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ON SEISMIC WAVE ATTENUATION IN AN HYDRATED CRUST

BY



PIERRE MICHEL ROULEAU

A THESIS SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND
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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled ON SEISMIC WAVE ATTENUATION IN AN HYDRATED CRUST submitted by PIERRE MICHEL ROULEAU in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY.

Abstract

Crustal seismic-wave attenuation in the form of $Q^{-1}(\omega)$ shows strong frequency (ω) dependence in the bandwidth .08 - 25 Hz. There are conjectured peaks near one Hz in several tectonic settings. The usual interpretation of this observation, elastic scattering arising from randomly distributed km-scale impedances in a single-layer crust, is not unique. I evaluate the viscous squeeze-flow process in fluid-filled mm-scale cracks characterising naturally rough fractures, in a layered crust. Significant absorption of seismic shear-energy propagating in the crust occurs by localised oscillatory viscous flow of compressible aqueous fluids and produces a robust peak which remains within the crustal-seismic bandwidth down to 20 km in an hydrated crust.

The demonstration is in four stages:

- 1) establishment of an approximate characteristic frequency of seismic dissipation by squeeze-flow, constrained by field-based measurements of rough-fracture topography and by geophysical manifestations of crustal hydraulic properties;
- 2) development of a generalised theory of compressible viscous squeeze-flow in a compliant low aspect-ratio crack, with explicit flow-regime dimensionless discriminants;
- 3) analytic solutions of linearised compressible-flow governing equations for squeeze-flow in a single idealised characteristic crack, with two sets of boundary conditions typical of rough fractures; and
- 4) scaling of the microscopic dissipation response of a single crack to the macroscopic km-scale response of fractured rock-formations, with theoretical mapping of intrinsic $Q^{-1}(\omega)$ and associated velocity dispersion $v(\omega)$ in a layered hydrated crust.

For a given crustal layer, $Q^{-1}(\omega)$ constrains characteristic crack geometry, fracture porosity, fractured-rock stiffness, and fluid viscosity and temperature. Such mapping gives the basic constituents needed to synthesise seismograms for a viscoelastic layered crust in order to interpret the measured $Q^{-1}(\omega)$ in terms of in situ crustal fluid properties. In addition to providing a new technique for further constraining the hydraulic character of the Earth's crust, maps of squeeze-flow Q^{-1} give a basis for examining the relative contribution of dissipative Q^{-1} and scattering Q^{-1} to measured attenuation.

Résumé

Le facteur de perte $Q^{-1}(\omega)$ des ondes sismiques guidées dans la croûte terrestre dépend fortement de la fréquence ω dans la gamme .08 — 25 Hz. L'atténuation de ces ondes apparaît maximale près de 1 Hz pour plusieurs provinces tectoniques. L'interprétation courante de cette observation fait appel à un mécanisme de diffusion élastique régi par les propriétés statistiques de contrastes d'impédance à l'échelle du km. Or cette interprétation, basée sur un modèle de croûte hétérogène à une seule couche, n'est pas unique. Je propose une explication alternative basée sur l'atténuation intrinsèque produite par l'écrasement de films fluides visqueux et compressibles saturant les épontes rugueuses de fractures naturelles. Les dimensions caractéristiques de l'espace poreux associé à ces fractures sont de l'ordre du mm. Je démontre que l'atténuation des ondes de cisaillement par ce mécanisme est significative pour un modèle de croûte stratifiée. L'écoulement local de fluides aqueux produit une atténuation qui dépend fortement de la fréquence d'oscillation et dont les maxima demeurent dans la gamme d'observation jusqu'à une profondeur de 20 km.

La démonstration est présentée en quatre étapes:

- 1) établissement d'une fréquence caractéristique du mécanisme d'absorption, estimée à partir de mesures topographiques de fractures naturelles et de levés géophysiques des propriétés hydrauliques de la croûte;
- 2) développement d'une théorie généralisée de l'écoulement oscillatoire d'un fluide visqueux compressible dans un pore mince, avec critères d'évaluation des régimes possibles d'écoulement;
- 3) résolutions analytiques des équations linéarisées gouvernant l'écoulement dans un pore idéalisé pour deux ensembles de conditions aux limites typiques d'une fracture à épontes rugueuses;
- 4) établissement d'un lien entre l'atténuation microscopique associée à un pore caractéristique et l'atténuation macroscopique à l'échelle des formations rocheuses, conduisant à une estimation du facteur de perte intrinsèque $Q^{-1}(\omega)$ et de la dispersion de la vitesse $v(\omega)$ dans une croûte stratifiée.

Pour une couche donnée, $Q^{-1}(\omega)$ reflète la géométrie d'un pore caractéristique, la porosité de fracture, l'impédance mécanique de la formation, la viscosité du fluide interstitiel, et la température ambiante. La distribution en profondeur du facteur $Q^{-1}(\omega)$ fournit les éléments nécessaires pour synthétiser des sismogrammes caractérisant une croûte viscoélastique stratifiée, ce qui permet d'interpréter l'atténuation observée en termes des propriétés in situ des fluides de la croûte. En plus de fournir une nouvelle technique d'étude du caractère hydraulique de la croûte terrestre, une telle distribution permet également d'examiner la contribution relative des mécanismes de diffusion élastique et de dissipation à l'atténuation observée.

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Chapter-1: Introduction

Earth materials are intrinsically dissipative as evidenced by the universal observation that seismic-wave motion inevitably ceases. Seismic dissipation is an important source of information on the composition, state, pressure, and temperature in the Earth's interior (Jeffreys, 1976). Despite considerable efforts the subject remains poorly understood, in part because the Earth behaves nearly elastically at seismic frequencies, and in part because the relevant dissipative mechanisms by which seismic energy is transformed into heat are difficult to identify. Interpretation of attenuation measurements in terms of dissipation is further complicated by seismic-wave scattering from randomly distributed impedances. In the past decade work on scattering has dominated attempts to interpret attenuation data for the lithosphere. The relative contribution of dissipation and scattering remains uncertain, especially with respect to their individual dependence on frequency.

This thesis aims at providing basic tools to interpret observations of seismic-wave attenuation in the Earth's crust, in terms of a contributory dissipative mechanism likely to predominate in fractured rock-formations saturated with a viscous fluid phase. This viewpoint is motivated by mounting evidence on the state of hydration of the crust (Bredehoeft & Norton, 1990) and by the characteristic strong frequency dependence of attenuation observed both in saturated rock-samples (Jones, 1986) and in the field (Sato, 1990).

1.1 Seismic attenuation problem: definition and background

Intrinsic attenuation generated in sinusoidally stressed Earth materials is quantified by the quality factor Q of a mechanical system by

$$Q^{-1}(\omega) = -\frac{1}{2\pi} \frac{\Delta \Sigma}{\Sigma_p} \quad [1.1]$$

where ω is angular frequency, Σ_p peak strain-energy stored in a unit volume per cycle, and $\Delta \Sigma (< 0)$ energy dissipated during this cycle (Aki & Richards, 1980).

Intrinsic attenuation is best denoted by Q^{-1} (i-e inverse Q) in order to reflect the fact that at the surface of a structurally heterogeneous body the composite attenuation eventually results from a weighted average of Q^{-1} 's and not of Q 's (Jackson & Anderson, 1970). Absorption, damping, internal friction, and dissipation are synonyms of intrinsic attenuation. The term dissipation is preferred here, with the understanding that it reflects irreversible removal of energy from the elastic deformational field. This removal is to be distinguished from that due to elastic scattering by random heterogeneities for which energy is not "lost" but merely redistributed in space and time.

These two energy-reduction mechanisms — dissipation and scattering — are fundamentally different. The former originates from microscopic material flaws, the latter from material contrasts nearing the macroscopic scale of wavelengths associated with the applied sinusoidal stress

(Wennerberg & Frankel, 1989). Q^{-1} in the Earth is by necessity estimated from surface measurements and so is a macroscopic quantity. Given the likelihood of scattering in a medium as heterogeneous as the Earth's crust, it is often questionable whether a Q^{-1} -estimate indeed reflects dissipation and not one smeared by a scattering contribution to $\Delta \Sigma$ in Eq-1.1.

The problem of seismic attenuation research can be formulated in two parts: 1) to map dissipative Q^{-1} in the Earth from the measured attenuation at its surface; and 2) to identify the parameters controlling the microscopic mechanisms by which strain-energy contributes to an entropy increase whose manifestation is macroscopic. The problem of separating dissipation from scattering in attenuation measurements belongs to the first part and is currently the object of great activity (e.g. Wennerberg, 1993). In this thesis, however, the primary concern is the second part, which confines the discussion within the generic topic of "physics of seismic dissipation" as it applies to crustal thermodynamic conditions.

While Q^{-1} is unequivocally defined by Eq-1.1, its interpretation from seismic measurements is fraught with assumptions made about seismic-waves, whose generation process and propagating medium properties are generally uncertain. The most fundamental assumption is that the response of the Earth to seismic stressing is linear, from which follows the mathematically convenient principle of superposition. Because this assumption is subject to confirmation and to qualification, the physical interpretation of a seismic Q^{-1} -estimate is not immediately transparent. The meaning of linearity is also adumbrated for a medium in which both dissipation and scattering simultaneously operate.

I discuss below the linearity assumption by drawing from poorly-cited but solid evidence supporting linearity and outline the framework of the theory of linear viscoelasticity within which seismic dissipation can be quantified and interpreted at both macroscopic and microscopic scales. This serves two purposes: 1) to clarify the physical meaning of Q^{-1} ; and 2) to set the scope of this thesis relative to current knowledge about seismic dissipation in the Earth as a whole.

1.2 The linearity assumption

Prior to formulating a clear interpretation of Q^{-1} , it is wise to recognise the results obtained by nearly a century of experimental seismology. Seismic-waves do propagate over large distances, an observation well-described by treating the Earth as being linearly and perfectly elastic (i-e non-dissipative). It is equally indisputable that seismic motion eventually ceases, an observation which can only be explained if the Earth is anelastic (i-e dissipative), although the degree of anelasticity must be small. The smallness of seismic-energy dissipation suggests that a linear wave-equation perturbed to first order may suffice for describing the strain-response of the Earth (cf. Ben Menahem & Singh, 1981). It also indicates that the elastic state can be taken as a reference about which the anelastic Earth slightly departs. However, the smallness of seismic dissipation does not *a priori* reject non-linear behaviour of the Earth's Q^{-1} response. This can be mani-

fested: 1) by a dependency on strain-amplitude; and 2) by a partial energy transfer from the driving frequency to its harmonics, even if Q^{-1} is independent of strain-amplitude (cf. Knopoff & MacDonald, 1960).

With this in mind, an informative expression for Q^{-1} is obtainable from Eq-1.1 by writing the strain-energy density stored in a cyclically stressed medium as

$$\Sigma = \int_0^1 \sigma(\epsilon) d\epsilon \quad [1.2]$$

where Σ is equal to the work done on the material by an externally applied stress σ which depends on strain ϵ , with both σ and ϵ normalised to their elastic counterparts (cf. Agnew, 1981). For a linearly elastic medium the function $\sigma(\epsilon)$ plots as a straight line in the (σ, ϵ) plane, whether stress increases or decreases; such non-dissipative case is described by Hooke's law for which Eq-1.2 yields the reference peak strain-energy $\Sigma_p = 1/2$. For a dissipative medium, energy is continuously absorbed during the cycling, producing an hysteresis loop in the (σ, ϵ) plane; in such case Eq-1.2 becomes a surface integral whose enclosed area $A = -\Delta \Sigma$, giving $Q^{-1} = A/\pi$ (cf. Stacey et al., 1975). Thus, the area of a stress-strain hysteresis loop characterises dissipation. Since hysteresis also follows from a lag of strain behind stress, Q^{-1} acquires a clear physical interpretation in that it reflects the amount of work done against internal friction.

This interpretation of Q^{-1} has been used by a few theorists and experimentalists (e. g. Stacey et al., 1975) in order to investigate the rheological regime of sinusoidally stressed rocks which is characterised by the shape of the loop, being elliptical if linear, and cusp-ended if non-linear. Initial reports of hysteresis observations from a variety of dry rock-samples for strains $> 10^{-6}$ showed cusped loops; this momentarily brought the linearity assumption under severe questioning. Later observations for smaller strains indicated elliptical loops and supported linearity (see Toksöz & Johnston, 1981). These results clearly demonstrate that Q^{-1} does depend on strain-amplitude and so call for non-linearity; however, their relevancy to seismic dissipation must be gauged against strains typical of seismic waves. Although strains generated near a seismic source much exceed 10^{-6} (Minster & Day, 1986), they quickly reach typical values of $\sim 10^{-8}$ at distances larger than a few characteristic source-dimensions. Since the latter is the commonest situation when seismically estimating Q^{-1} , non-linear effects directly associated with strain-amplitude can be expected to be small. Independent measurements of Q^{-1} from the decaying response of resonating rock-samples, dry and saturated, confirm the amplitude independency for strains smaller than 10^{-6} (Winkler, Nur & Gladwin, 1979). For these the characteristically non-linear Coulomb friction has been shown inoperable (Mavko, 1979).

It may be argued that the above laboratory observations represent but a poor sampling of Earth materials and do not preclude non-linear effects of the second type, i.e. energy transfer to higher harmonics, without strain-

amplitude dependency. However, field-based evidence for linear dissipation in the Earth does exist and is significant. The evidence is from studies of Earth-tides whose relevancy to the seismic attenuation problem is rarely acknowledged. Although tidal frequencies are much lower than seismic ones, the fact that tidal strains are of the same order of magnitude as typical seismic strains, combined with the very high signal-to-noise ratios obtainable from long time-series of tidal observations, provides a good test for the linearity of the seismic strain-response of the Earth. Tidal frequencies are inherently discrete and accurately known from astronomy (unlike the seismological situation); thus, if there were significant non-linearities in the Earth, they would generate strain signals at higher harmonics of the forcing frequencies.

Agnew (1981) performed this linearity test using tidal strain data of unsurpassed signal-to-noise ratios, together with realistic models for non-linear dissipation characterised by cusped stress-strain hysteresis loops. Although non-linearities were observed at the harmonics predicted by the models, these were traced back to loading strains from non-linear marine tides, leaving negligible higher-harmonic energy upon their removal. Agnew's result applies to seismic-dissipation proper because tidal deformations are insensitive to scattering. This is an important observation which also applies to the laboratory and stress-strain hysteresis studies cited above.

Therefore, it can be inferred that the dissipative Earth does behave linearly at the level of seismic straining and that non-linear intrinsic mechanisms, even if active, have negligible effects on the observable strain-response. Yet, seismic dissipation is observed; therefore, it must arise from linear processes. This conclusion has been reached independently from the usual and somewhat convoluted argumentation based on purely seismological considerations (cf. Aki & Richards, 1980). Thus, the linearity assumption has solid support and so can confidently be exploited to interpret Q^{-1} for strain-energy transmitted by way of linear waves.

1.3 Physical interpretation of seismic Q^{-1}

In practice, Q^{-1} is estimated from observations of the spatial or temporal decaying of seismic waveforms, the amplitude of which is proportional to the square-root of energy for linear waves. This latter attribute leads to the interpretation that $Q^{-1}(\omega)$ estimated from a spatially decaying wave of fixed frequency gives the rate at which energy is dissipated per wavelength of distance travelled, and that Q^{-1} estimated from a temporally decaying waveform gives the fractional energy loss per time increment (for a demonstration and other wave-related interpretations see Ben-Menahem & Singh, 1981). The interpretation for spatially decaying waves is clear when the observed amplitude is characterised by a single frequency; this is an uncommon occurrence in seismograms where only a few signals have at best one predominant frequency. However, spectral analysis of any seismic waveform keeps intact the physical meaning of $Q^{-1}(\omega)$, attributing to each spectral component its own attenuation value, which then characterises dissipation at that particular frequency.

The single-frequency nature of $Q^{-1}(\omega)$ is emphasised here because it is still assumed, based on an antiquated review (Knopoff, 1964), that $Q^{-1}(\omega)$ is effectively frequency-independent. While $Q^{-1}(\omega)$ often appears constant over wide frequency bands, the ensemble of modern attenuation data do require dissipation to be frequency-dependent in the Earth (Anderson & Given, 1982; Clements & Abdul-Rahman, 1988; Karato & Spetzler, 1990). This is especially so for the lithosphere where the measured attenuation above one Hz is significantly less at higher than lower frequencies over a single decade, an observation which can hardly be explained by a weighted average of frequency-independent Q^{-1} 's (Herraiz & Espinosa, 1987). With respect to laboratory data, $Q^{-1}(\omega)$ indeed appears independent of frequency but only for dry rock at temperatures much lower than sub-solidus; at higher temperatures $Q^{-1}(\omega)$ exhibits significant frequency dependence (Jackson, 1986). $Q^{-1}(\omega)$ for fluid-saturated rocks is strongly frequency-dependent within two frequency decades, with peak values much higher than for dry conditions regardless of rock-type (Jones, 1986).

Although the linearity assumption leads to a clear interpretation of $Q^{-1}(\omega)$ for dissipated linear seismic-waves, there remains the problem of estimating the possible contribution of scattering to the attenuation observed at the surface of a composite body such as the Earth's crust. The attenuation attributes of the crust are in a large part characterised by the decay-rate of high-frequency coda-waves

($\frac{\omega}{2(\pi)} > 1\text{ Hz}$) which appear as successive arrivals of

decaying amplitudes and random phases on seismograms, most often beyond the onset of the primary S-wave. Some form of scattering is required to explain the long duration and robust character of these coda-waves which are ubiquitously observed within a radius of 100 km or so of an epicentre (Herraiz & Espinosa, 1987). Much of the recent work on scattering has concentrated on proportioning the respective contribution of dissipation and scattering to the attenuation estimated from the decaying envelope of coda-waves (Wennerberg, 1993). A rich variety of scattering theories have been developed to this end; however, there is no emerging consensus on a reliable proportioning. It is generally assumed that scattering and dissipation are additive which implies linearity.

At issue here is whether that part of the observation due to scattering is linear. There is no evidence supporting linearity for scattered seismic-waves other than an internal consistency between most observed coda-wave properties and predictions from linear scattering models. Thus, dissipation and scattering each seems to be linear, but it is conceivable that their interaction is not simply additive (e. g. Beltzer et al., 1989). However, experimental and theoretical investigations of the additivity assumption indicate that it is an excellent approximation inasmuch as the combined attenuation is much smaller than unity (e. g. Dubendorff & Menke, 1986; Beltzer et al., 1989). Since reported attenuation estimates for the lithosphere have a typical range $\frac{1}{1000} < Q_{\text{Obs}}^{-1} < \frac{1}{100}$, it is reasonable to expect that dissipation and scattering are

additive; it remains to be shown which linear combination best reflects the phenomenon. Evidence is mounting that the decay rate of coda-waves is dominated by dissipation and not scattering as commonly thought (Frankel & Wennerberg, 1987; Wennerberg, 1993). If this is so, then even if there were non-linearities in the scattering part of the observation they would have small effects on the total attenuation estimated from coda decays.

In summary, there is firm evidence that seismic-wave dissipation results from linear processes, and circumstantial evidence that seismic-wave scattering and its interaction with dissipation are linear processes as well. This result fully justifies use of a linear mathematical framework, not only for extracting Q^{-1} from seismic-waves propagating in a dissipative stochastic medium, but also for inferring the mechanisms by which strain-energy is dissipated. With respect to the latter, a constitutive relation between stress and strain which accounts for both linearity and dissipation must be invoked in order to determine the material parameters which control the irreversible energy-loss manifested by attenuated seismic-waves.

1.4 Dissipation mechanisms and the theory of linear viscoelasticity

In order to infer rock properties from seismic-wave data it is necessary to specify a constitutive law relating stress to strain. The commonest law used is that characterising a perfectly elastic medium for which stress is an instantaneous and linear function of strain, with a proportionality constant (i-e modulus) set by the mechanical properties of the medium. This law cannot account for the cessation of wave-motion caused by dissipation, an additional intrinsic property of Earth materials. The character of dissipation together with the nearly elastic state of rocks are embodied in the constitutive law governing linear viscoelastic materials, for which the strain-response is the sum of two parts, one instantaneous, the other time-delayed and associated with a relaxation time during which energy is dissipated.

The theory of linear viscoelasticity provides a mathematically and physically sound framework for characterising the rheological behaviour of the dissipative Earth, both at the macroscopic and microscopic scale (for details, including thermodynamics, see Minster, 1980). Mathematically, the strain tensor is expressed as a time-integral convolving the stress tensor with a creep-rate tensor, thereby establishing a linear relationships between strain, stress and stress-rates, with coefficients set by material properties. This convolution integral embodies Boltzmann superposition principle which secures the attributes of linearity, single-valuedness, continuity, time-invariance, and causality (for a demonstration, see Brennan & Smylie, 1981). For an isotropic medium, the tensorial symmetries permit separation of the strain-field into an isotropic part and a deviatoric part, related to their complementary stress-field by a viscoelastic bulk modulus and shear modulus, respectively. Ben-Menahem & Singh (1981) show that combining Boltzmann's integral law with Cauchy's equation of motion leads to a Navier equation with time-dependent moduli. This is the expression of the constitutive law of a generalised linear vis-

coelastic medium. Thermodynamic constraints require that relaxed moduli (i.e. as time tends to infinity) are lower than the instantaneous or unrelaxed moduli (Gurtin & Herrera, 1965).

The Fourier transform of the viscoelastic Navier equation is isomorphic to its elastic complement. The only difference is that the viscoelastic moduli are complex. The ratio of the imaginary-to-real part of these moduli, which takes various forms depending on loading, can be interpreted as a phase lag of strain behind stress and so gives a measure of dissipation. Thus, dissipation can also be written

$$Q^{-1}(\omega) = \frac{M_{imag}(\omega)}{M_{Real}(\omega)} = \tan \Phi(\omega) \quad [1.3]$$

where $\Phi(\omega)$ is the angle between the real and imaginary part of modulus $\bar{M}(\omega)$ in an Argand diagram. Eq-1.3 is equivalent to Eq-1.1 for the low Q^{-1} values typically encountered in seismology (O'Connell & Budiansky 1978), and for which $\tan \Phi(\omega) \approx \Phi(\omega)$. Hence, the physics of dissipation is amenable to description in terms of the imaginary and real parts of the viscoelastic moduli. This can be done for both microscopic and macroscopic scales with various degrees of complexity, from purely phenomenological (e. g. symbolic springs and dashpots) to elaborate characterisation by physics laws governing a loss mechanism (e. g. Murphy, Winkler & Kleinberg, 1986).

The correspondence between elastic and viscoelastic constitutive relations ensures that the solution of a viscoelastic problem is similar to its elastic counterpart (this result is referred to as "the correspondence principle"). Thus, in order to solve the wave-propagation problem to obtain the macroscopic response of dissipation, the viscoelastic spectral Navier equation can be separated into two Helmholtz equations, one scalar for irrotational (P) waves, the other vectorial for equivoluminal (S) waves (Ben-Menahem & Singh, 1981). The viscoelastic Helmholtz equation (scalar or vectorial) reads

$$\nabla^2 \Psi + [\bar{\kappa}(\omega)]^2 \Psi = 0 \quad [1.4]$$

where Ψ is the potential associated with either irrotational or solenoidal motion, and $\bar{\kappa}(\omega) = \omega / \bar{v}(\omega)$ is a complex wavenumber which depends on the wave velocity $\bar{v}(\omega)$ [bars denote complex quantities]. For P-waves,

$$[\bar{\kappa}_p(\omega)]^2 = (\rho \omega^2) / (\bar{k} + \frac{4}{3} \bar{\mu}) \quad [1.5]$$

and for S waves,

$$[\bar{\kappa}_s(\omega)]^2 = (\rho \omega^2) / \bar{\mu} \quad [1.6]$$

where ρ is density (taken real), and $\bar{k}$ and $\bar{\mu}$ the bulk and shear modulus of the viscoelastic material, respectively. Harmonic-wave solutions to Eq-1.4 are obtained by the *ansatz*

$$\Psi = \Psi_0 e^{i(\omega t - \mathbf{K} \cdot \mathbf{R})} \quad [1.7]$$

where t is time and $\mathbf{R}$ the position vector. $\mathbf{K}$ is a bivector

defined by $\mathbf{K} = \mathbf{P} - i \mathbf{A}$, where $\mathbf{P}$ is normal to surfaces of discontinuity of constant phase $\mathbf{P} \cdot \mathbf{R} = \text{constant}$, and $\mathbf{A}$ is normal to surfaces of constant amplitude $\mathbf{A} \cdot \mathbf{R} = \text{constant}$. The wave-amplitude vector $\mathbf{A}$ and the phase vector $\mathbf{P}$ are related to $\bar{\kappa}(\omega)$ by

$$[\bar{\kappa}(\omega)]^2 = [\kappa_{real} - i \kappa_{imag}]^2 = |P|^2 - |A|^2 - 2i|P||A| \cos \chi \quad [1.8]$$

where χ is the angle between $\mathbf{A}$ and $\mathbf{P}$, with $0 \leq \chi \leq \pi/2$ so that the wave-amplitude is bounded in the direction of propagation.

The set of equations 1.4→1.8 embodies three fundamental manifestations of viscoelasticity: 1) an intrinsic dispersion of wave-velocity, since $\bar{v}(\omega)$ is a function of frequency; 2) a diminution of wave-amplitude as distance increases, expressed by the factor $e^{-i \mathbf{A} \cdot \mathbf{R}}$ and 3) is an elliptical particle trajectory for highly inhomogeneous P and S waves, namely those associated with large values of angle χ (see Ben-Menahem & Singh, 1981). Of these, the first two are the most important for seismic-waves propagating in the crust and mantle. The third one becomes apparent only for dissipation of anomalously high magnitude.

Since the range of Q^{-1} values reported for the crust and mantle is small several simplifications of the formal results of linear viscoelasticity are possible. For these low values, $|A|^2 \ll |P|^2$ and $\chi \rightarrow 0$, which leads to

$$Q_s^{-1}(\omega) \equiv 2 \frac{|A|}{|P|} = \frac{\mu_{imag}}{\mu_{real}} \quad [1.9]$$

for S-waves, and for P-waves to

$$Q_p^{-1}(\omega) \equiv 2 \frac{|A|}{|P|} = \frac{k_{imag} + \frac{4}{3} \mu_{imag}}{k_{real} + \frac{4}{3} \mu_{real}} \quad [1.10]$$

All quantities are real and the labelling specifies dissipation in shear (Q_s^{-1}) or in dilation (Q_p^{-1}). The low loss approximation also permits to write the wave-velocity (P or S) as

$$v(\omega) \equiv \sqrt{\frac{M_{real}(\omega)}{\rho}} \quad [1.11]$$

and to define an attenuation factor, observable from spatially decaying waves, as

$$\gamma(\omega) \equiv \frac{\omega Q^{-1}(\omega)}{2v(\omega)} \quad [1.12]$$

Hence, measurements of P and S wave-velocities (also surface-waves) and associated attenuation factors as a function of frequency completely characterise the macroscopic seismic-response of the dissipative Earth. A further character of waves propagating in a linear viscoelastic medium arises from the physical requirement of causality, which relates $v(\omega)$ and $\gamma(\omega)$ via Hilbert transforms and equations like the Kramers-Krönig dis-

persion relations (for details, see Ben-Menahem & Singh, 1981). These relations are most useful in that only $v(\omega)$ or $\gamma(\omega)$ needs be observed for forwardly testing the validity of the dissipation mechanism attached to $Q^{-1}(\omega)$.

Given the layered structure of the Earth, it is desirable to expand the above wave properties by applying the correspondence principle. Then well-posed problems for layered viscoelastic structures can be formulated. For example, Lee & Solomon (1978) developed a technique in which dissipation/dispersion data pairs, together with a specified Kramers-Krönig relation, are simultaneously inverted for mapping $Q^{-1}(\omega)$ with depth. Their resolution is severely affected by noise. Forward techniques relying on restrictive assumptions have also been developed in which $Q^{-1}(\omega)$ functions are inferred from physical and laboratory constraints and then fitted to the surface Q^{-1} data (Anderson & Given, 1982).

Seismic dissipation at the surface of the Earth is observed with variable accuracy (5 — 30%) in discrete frequency bins, from free-oscillation eigenmodes (10^{-4} — 10^{-3} Hz) through surface-waves (10^{-2} — 10^{-1} Hz) to short-period body and crustal waves (10^{-1} — 10^2 Hz). A significant dispersion ($\sim 1.5\%$) is also observed across this spectrum, with velocity rising from low to high frequencies as would be expected from linear viscoelasticity. Dissipation in shear is generally larger than in dilation at least for frequencies less than a few Hz. Global estimates of dissipation in shear, obtained by interpolating through the six decades of the seismic bandwidth, shows a nearly constant $Q_s^{-1}(\omega)$ of $\sim 1/200$ from 10^{-3} to 10^{-1} Hz. Then $Q_s^{-1}(\omega)$ linearly decreases to $\sim 1/10000$ at 10 Hz

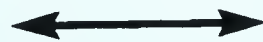
(cf. Minster, 1980). Such data, together with Q^{-1} estimates for P-waves, surface-waves, Earth tides ($\sim 10^{-5}$ Hz), and Chandler wobble ($\sim 10^{-8}$ Hz), have served as a basis for forward mapping of $Q^{-1}(\omega)$ in the Earth. This has led to a standard mapping for the depth-distribution of $Q^{-1}(\omega)$, portrayed as six wide absorption bands shifting about the seismic bandwidth as depth increases towards the Earth's liquid core (Anderson & Given, 1982).

The above-mentioned viscoelastic-wave attributes hold regardless of the causative dissipation mechanism and so cannot by themselves uniquely constrain the microphysics of dissipation. Additional constraints are required for relating these macroscopic quantities to microscopic-scale properties governing dissipation. Laboratory Q^{-1} studies together with arguments based on known physics, although only partially reproducing *in situ* conditions, offer the most useful constraints.

A large body of solid-state loss mechanisms is thought to be operative in the Earth, including atomic-scale point defects, dislocations, molecular kinks, grain boundary migrations, extrinsic and intrinsic diffusion, thermoelastic relaxation, etc., all of which are strongly sensitive to molar volume, temperature, pressure, and especially frequency (for a recent review, see Karato & Spetzler, 1990). Each of these mechanisms can be parameterised by a standard linear solid, a mechanical analogue characterised by a bell-shape frequency dependent $Q^{-1}(\omega)$ and associated modulus dispersion, as shown in figure-1.1. The maximum $Q^{-1}(\omega)$ is set by the modulus defect, the difference between the modulus values at low and high frequency, and occurs at a frequency characteristic of the

Dissipation mechanism analogue

standard-linear solid



dissipation/dispersion pair

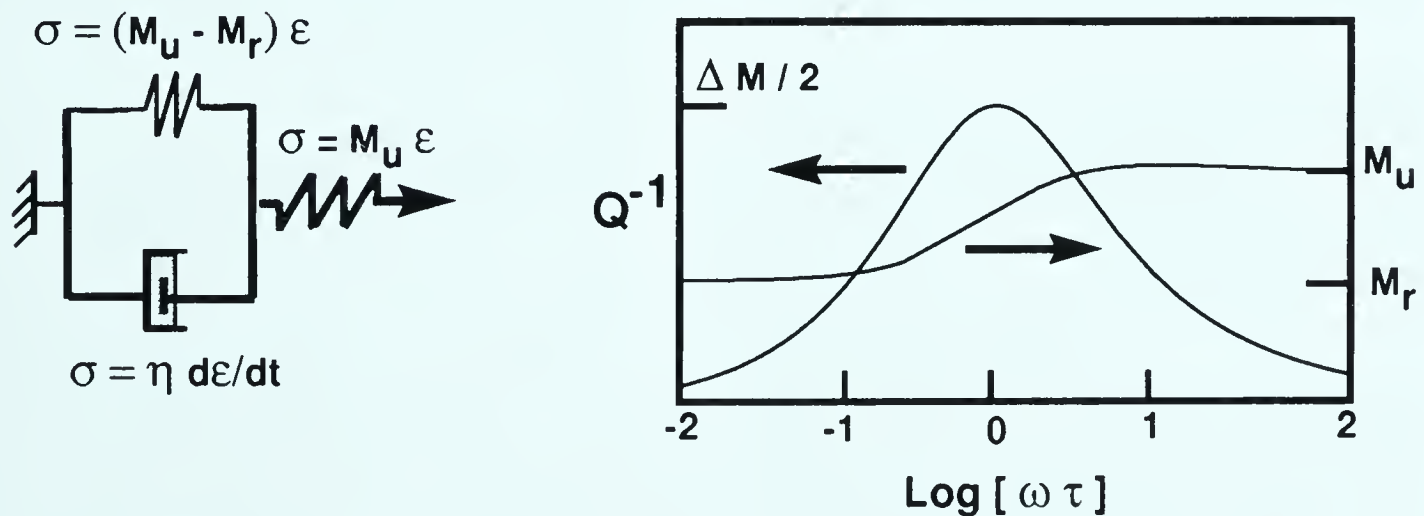


Figure-1.1

LEFT: a representation of a standard-linear solid as a three-component mechanical analogue for a dissipation mechanism characterised by a single relaxation time τ . Each component relates stress $\sigma(t)$ to strain $\epsilon(t)$ as shown, with M_u and M_r the value of the real part of viscoelastic modulus M at instantaneous (unrelaxed) and infinite (relaxed) time, respectively. The total strain-response is obtained by summing, time dependence enters via the dashpot of viscosity η .

RIGHT: dissipation (Q^{-1}) and modulus dispersion response to harmonic stressing, as a function of frequency normalised to relaxation frequency. Dissipation and dispersion relate to the imaginary and real part of M , respectively, with both parts linked via a Kramers-Krönig relation. Maximal Q^{-1} occurs when the driving frequency equates the relaxation frequency, with magnitude $= \Delta M / 2$. $\Delta M = M_u - M_r$

relaxation time during which the mechanism is activated. This spring-and-dashpot analogue is quintessential of the theory of linear viscoelasticity and can describe the phenomenology at both microscopic and macroscopic scale.

Given the intrinsically heterogeneous nature of rock materials and the wide range of thermodynamic conditions in the Earth, the occurrence of many of the proposed mechanisms is likely. In support of this view is the apparent success of Anderson & Given's (1982) Q^{-1} model, whose six absorption bands are nearly flat over an octave in frequency, a result consistent with the superposition of a many mechanisms activated at various frequencies. The approximate frequency independence of $Q^{-1}(\omega)$ in the Earth is (Anderson & Given 1982) more a reflection of the paucity and limited precision of data than a firm statement about the physics of dissipation. Nevertheless, the multitude of possible mechanisms poses an apparently insuperable problem for characterising the physics of dissipation in any useful detail, except perhaps at either end of an absorption band.

Despite this inherent difficulty, important clues about microscopic processes are apparent from the observed dissipation of seismic-waves. Since $Q_s^{-1}(\omega)$ is generally larger than $Q_p^{-1}(\omega)$ dissipation preferentially occurs in shear and suggests discarding bulk-modulus controlled mechanisms (e. g. thermoelastic relaxation). The seismic $Q^{-1}(\omega)$ global spectrum exhibits fine structure, especially for surface-waves which reflect more regionalised estimates of dissipation (e. g. Jackson, 1970; Lee & Solomon, 1978; Aki, 1980). Of particular interest is the significantly frequency-dependent attenuation ubiquitously observed in the sub-band 0.05 to 25 Hz, although scattering in this high-frequency range poses contamination problems (it is then prudent to employ the general term of "attenuation" instead of the specific term "dissipation").

The attenuation observed at high frequencies is intimately linked to shear-waves confined to the lithosphere and suggests the presence of a peak near 1 Hz about which attenuation decreases over less than three decades, akin to a standard-linear solid (cf. Aki, 1984). If such frequency signature is not due to scattering but to dissipation, then there is hope to characterise the causative mechanism in some details, especially when considering that the range of thermodynamic conditions in the lithosphere is much smaller than for the whole Earth.

Thus, it seems promising to concentrate efforts on dissipation in the lithosphere and to search for a realistic causative process. However, satisfactorily linking the macroscopic attenuation response of the lithosphere to a microscopic dissipative process, even if it is manifested by a narrow range of relaxation frequencies, is still a problem. Apart from identifying the mechanism itself, several inescapable aspects must be addressed, including scattering effects, layering, scaling, temperature, pressure, rheology, dynamic regime, and the identification of both macroscopic and microscopic properties shared by many tectonic environments.

With respect to identifying the mechanism itself, laboratory Q^{-1} data obtained under crustal conditions offer some clues in that a robust, strong frequency-dependence is observed over a narrow frequency range in all types of

fluid-saturated rocks (Jones, 1986). The causative mechanism has been traced to localised oscillatory viscous flow of interstitial fluids in the narrowest intergranular space, a process called squirt-flow and modelled in some detail by Murphy et al. (1986).

This observation suggests the predominance of a single mechanism, clearly related to interstitial fluid properties and rock micro-deformability. Since the Earth's crust appears to be pervaded by fluids (Bredehoeft & Norton, 1990), this laboratory-inferred mechanism may be efficient in the crustal-seismic band [0.08 — 25 Hz]. However, there are major stumbling blocks for transposing this mechanism to field conditions, chief among which is the fact that its characteristic frequency falls in the kHz range and so far removed from the seismic band of interest. Moreover, it has been suggested that the microstructure conducive to squirt-flow results from a bias due to coring common to all rock-samples (e. g. O'Connell, 1984). Also, the most developed squirt-flow model (Murphy et al., 1986) has a weak mathematical and physical foundation.

Despite these apparent obstacles, it may be possible, in light of the largely untapped constraints provided by recent data on the state of hydration of the crust, to establish squirt-flow as a viable and predominant mechanism for seismic dissipation in large portions of the Earth's crust.

With regards to correctly modelling the process of squirt-flow, it might seem logical to rather apply the widely used Biot theory of wave propagation in porous media. This has a clearer mathematical physics footing and also predicts a similar dissipation by involving bulk and not localised viscous fluid-flow (Biot, 1956). However, the peak dissipation predicted by Biot's theory almost always falls at much higher frequencies than that predicted by squirt-flow (Rasofolao, 1991) and is therefore even more remote from the lithospheric seismic-band. This arises from the implicit assumption in Biot's theory that field-variables can be characterised by globally averaging over a representative space shared by pore-fluid and solid-matrix materials. This disregards exactly those localised variations which generate squirt-flow.

The basic premiss of squirt-flow is that the macroscopic sinusoidal stress-field must differentially and significantly localise at pore boundaries so that viscous flow may absorb part of the driving energy. This is an assumption which, although consistent with ensuing results, has no direct experimental support since strain-gauges of rock-grain scale are impracticable. Here again, Earth tides observations provide a favourable demonstration relevant to seismic-dissipation.

It has been amply demonstrated (e. g. Baker, 1980; Rouleau, Jones & Rogers, 1988) that tidal strain and tilt meters measure a large part of the local deformation about the immediate instrument setting, despite the enormous difference between site dimension and tidal wavelength (e. g. 60 cm gauge versus a fraction of the Earth's circumference for tides). For instance, the magnitude of localised deformations observed by collocated tiltmeters in a shaft of irregular geometry can range $\pm 60\%$ of the input tidal signal over distances of a few tens of m. Thus, it can be inferred that the stress associated with passing seismic-waves of long wavelengths indeed locally gener-

Search for an Explanation of Observed $Q_t^{-1}(\omega)$ at High Frequencies

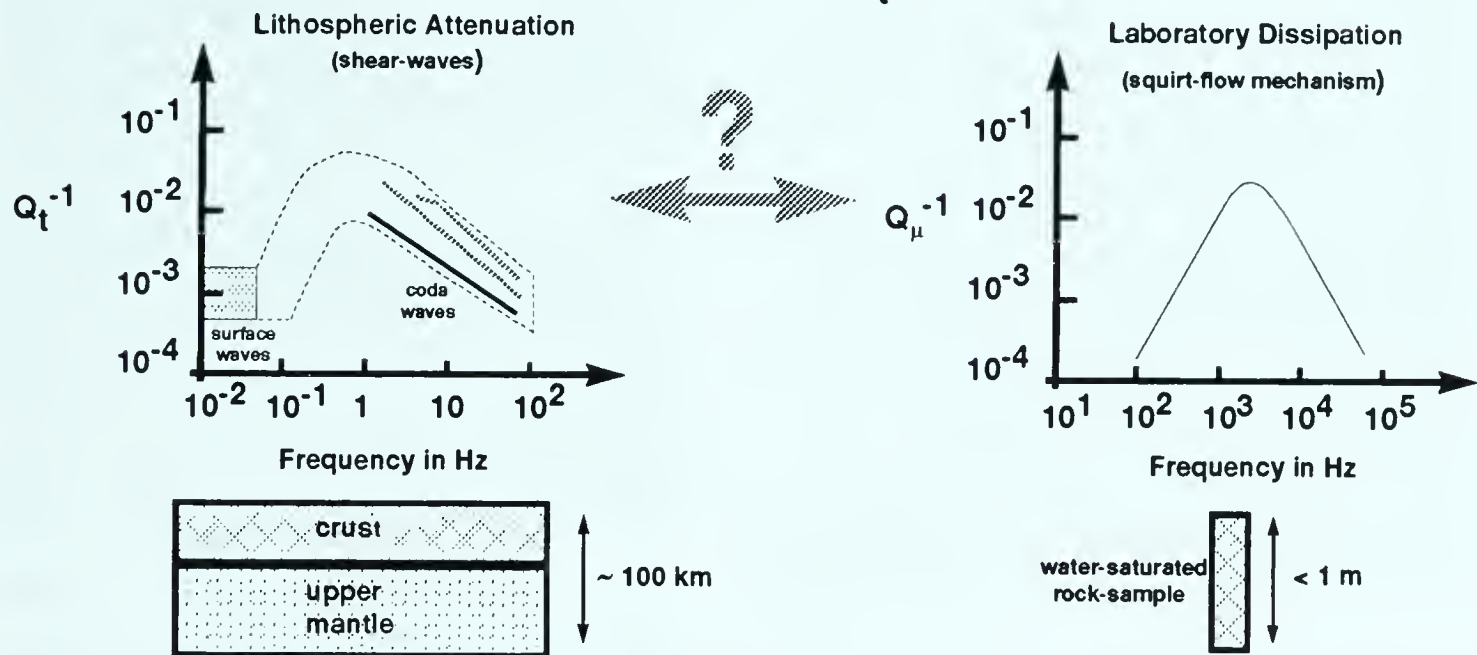


Figure-1.2

LEFT: schematic diagram of attenuation $Q_t^{-1}(\omega)$ measured from shear-waves in the high-frequency portion of the Earth seismic-band. Hatched box shows Q_t^{-1} values for surface-waves, full lines for coda-waves, and dotted lines range of values, with conjectured characteristic frequency near 1 Hz, after Aki (1984). Waves from which Q_t^{-1} is estimated are confined to the lithosphere (below).

RIGHT: schematic diagram of dissipation in shear Q_{μ}^{-1} measured in water-saturated rock-samples of dimension shown (below). Characteristic frequencies fall near 1 kHz for all rock-types, after Jones (1986). In this work, a relationship is sought between the two frequency signatures depicted, with the aim of constraining in situ fluid-rock properties from attenuation observations.

ates significant differential deformations wherever small-scale corrugated fluid/rock boundaries occur.

1.5 Thesis description

The above considerations circumscribe a problem of seismic dissipation research which consists in finding a consistent physical interpretation for the attenuation observed in the high-frequency portion of the Earth seismic-band, characteristic of the lithosphere. Provided a rapprochement is first made between the frequency signature of lithospheric attenuation and that typical of the laboratory-inferred squirt-flow mechanism, the physics of seismic dissipation in the lithosphere can, in principle, be described in useful detail within the framework of the theory of linear viscoelasticity and of viscous hydrodynamics.

This thesis addresses the following basic question, illustrated in figure-1.2:

Can the bell-shaped frequency-dependent seismic attenuation reported for the lithosphere result from preferential dissipation by viscous flow of interstitial fluids with characteristic frequencies within the band of high-frequency seismic-waves?

The answer is yes, at least for the upper crust. This leads to a new tool for interpreting seismic $Q^{-1}(\omega)$ and a way to constrain further the hydrated state of the Earth's crust. The thesis demonstrates this result in four papers. The general philosophy is to model the loss mechanism at the microscopic scale first and then to synthesise its effect at the macroscopic scale.

Paper-1

In the first paper (chapter-2), after combining reported attenuation measurements for an active and passive tectonic setting at frequencies ranging from 0.08 to 20 Hz, I present arguments against the apparent success of a typical scattering-based explanation and suggest a new approach for a dissipation-based explanation. This explanation reconciles the discrepancy between the frequency-dependence signature of crustal attenuation, characterised by a peak value near one Hz, and that of rock-samples, characterised by peak values in the kHz range. Central to the argumentation is an estimated characteristic frequency associated with maximal dissipation by oscillatory motion of a crack-filling viscous fluid without bulk flow (i.e. squirt-flow). I show this to be likely in naturally fractured rock when operating within the rough interspace of crustal fractures. This frequency is approximately one Hz for low aspect-ratio cracks representative of rough fractures and whose characteristic nature and geometry are constrained by field-based measurements and manifestations of crustal hydraulic properties. This result justifies a full assault on the problem of quantifying dissipation by viscous fluid-flow in the basic structural element characterising naturally fractured-rock: a single compliant crack of low aspect-ratio.

Paper-2

In the second paper (chapter-3), I clarify the physics of viscous losses in a low aspect-ratio saturated crack undergoing oscillatory squeezing, and develop an encompassing theory for the pressure-driven motion of a compressible viscous fluid in such crack with arbitrary crack-wall topography. The ensuing mechanism is referred to as "squeeze-flow". The theoretical develop-

ment is based on gas-lubrication theory which I extend, from first principles and with a minimum of assumptions, to slightly compressible fluids in the liquid state. The essential result is a dimensionless, non-linear, integro-differential governing equation for the thickness-integrated viscous squeeze-flow of a surface-adhering fluid-film, from which all squirt-flow models proposed so far can be deducted.

This governing equation can be used to study non-linear attenuation, a field of growing interest [Day & Minster, 1992] but not explored here, and, more importantly, to discriminate possible flow regimes by simple dimensionless groupings. Two of these groupings, squeeze-flow Reynolds number R_s and squeezing number σ , combine crack dimensions, driving frequency, fluid viscosity, and ambient pressure, and so provide unequivocal ways to evaluate inertial and fluid compressibility effects at crustal thermodynamic conditions.

Paper-3

In the third paper (chapter-4), I apply the theory by first showing the importance of fluid compressibility and the negligibility of inertia for viscous squeeze-flow within seismically-stressed, water-saturated cracks characterising crustal fractures. This considerably simplifies the compressible squeeze-flow governing equation. Further reduction follows from modelling a fracture-characterising crack by two coaxial, impermeable disks separated by a thin fluid-film. Invoking the smallness of seismic strain and assuming parallelism of disk-walls, oscillatory pressure-flow of the constricted film obeys a cylindrical-coordinate, inhomogeneous, linear, differential equation of the Bessel type. I present analytical solutions of this equation for two sets of boundary conditions at the radial periphery of the fluid-film, written, respectively, for a porous-rigid ring and for a permeable-deformable ring, both rings of finite extent. The latter set requires an additional governing equation for compressible pressure-flow in a deforming permeable medium, which reduces to a diffusion equation for conditions of interest. Solutions are expressed in terms of a complex, frequency-dependent, characteristic-crack stiffness, whose imaginary and real parts quantify seismic dissipation via the lag of fluid motion behind crack-wall squeezing. For relevant parameters, results show a significant, frequency-selective dissipation, with peak values remaining in the crustal-seismic bandwidth, down to at least mid-crustal thermodynamic conditions.

Paper-4

In the fourth paper (chapter-5), I synthesise the microscopic-scale dissipation operating in a single characteristic crack to the km-scale of saturated, fractured-rock formations pervaded by such cracks. Within the framework of a conceptual model for a layered and porous crust, I justify the use of an effective-medium, sphere-pack model formulated by Murphy et al. (1986) for the laboratory-scale viscoelastic moduli of saturated, granular materials. I summate model parameters from field-based constraints on the geometry of crustal heterogeneities and from seismic inferences on the composition and porosity of crustal layers. The large-scale response of dissipation by localised squeeze-flow in fracture-characteris-

ing cracks is expressed in terms of the loss factor $Q_s^{-1}(\omega)$ for shear-waves and associated dispersion $v(\omega)$, as a function of frequency. An order-of-magnitude mapping of the depth-distribution of seismic (Q_s^{-1} , v) pairs in a three-layer crust is presented for a schematic hydrated continental crust environment. The result shows that significant peak values of $Q_s^{-1}(\omega)$ remain within the crustal-seismic bandwidth down to mid-crustal levels, in contrast with predictions from Biot's theory of wave-propagation in permeable media. I also show that dissipation by local fluid-flow is unlikely for the lower crust whereas scattering attenuation by km-scale lenticular impedances may have some significance.

Chapter-6 summarises the results and their implication for interpreting the observed apparent crustal $Q^{-1}(\omega)$ in terms of *in situ* fluid-rock properties. Several lines of further research using investigative techniques developed in this thesis are suggested

Nota

Seismic dissipation is estimated at the surface of the Earth and it is customary to label Q^{-1} with the wave type used, for example Q_p^{-1} for dilatational waves, Q_s^{-1} for shear waves, Q_c^{-1} for coda waves, Q_k^{-1} (bulk dissipation) for spheroidal normal-modes, etc. If the medium is homogeneous, only the notation Q_s^{-1} and Q_k^{-1} is directly related to dissipation since the stress-strain field is intrinsically separable into an isotropic part and a shear part. For other wave-types the propagation process must be related to these two fundamental parts, prior to interpreting dissipation. The theoretical calculations presented here are developed in terms of the real and imaginary part of an effective shear modulus for single, macroscopically homogeneous layers of lossy materials and so bear a direct relationship to intrinsic attenuation therein.

All seismic " Q^{-1} " measurements are a composite expression of "attenuation" occurring throughout that part of the heterogeneous Earth actually sampled by the waves used in the estimation. This, too often, is not clear in the seismic-attenuation literature, especially for high-frequency waves which comprise a rich spectrum of wave-types (the regional phases), all subject to scattering. In the presence of scattering it is appropriate to use Q_{sc}^{-1} for that part of the measurement thought to result therefrom.

Richards & Menke (1983) pointed-out that this lack of rigour has led geophysicists adrift from the seismic-attenuation problem. Ironically, they have contributed to the confusion by calling scattering Q_{sc}^{-1} "apparent" Q_a^{-1} , a term traditionally used for the observed attenuation, i-e that which "appears" at the surface of the heterogeneous Earth. To avoid misunderstanding, the measured attenuation will be referred to here as total Q^{-1} , labelled Q_t^{-1} , with the understanding that it reflects a compounding of spatially distributed dissipation and of scattering. Dissipation proper will sometimes be denoted by Q_i^{-1} where "i" stands for intrinsic attenuation (i-e dissipation), otherwise the simple Q^{-1} is used.

1.6 References

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Chapter-2: Seismic Wave Attenuation in an Hydrated Crust: Conditions for an Intrinsic Mechanism¹

The Earth's crust is pervaded by H₂O-rich fluids in sufficient amount to significantly affect its response to a wide range of thermo-mechanical stresses (cf. Fyfe, Price & Thompson, 1978). It is thus necessary to account for the large-scale flow of water through at least the upper crust in order to model the evolution of geodynamical systems (Mase & Smith, 1987). However further advance of such modelling requires constraints on the hydraulic properties of the crust at the hundreds of km scale, particularly on compressibility and permeability. The lack of constraints partly arises from the difficulty of recognising the signature of fluids in geophysical data pertaining to the body of the crust. It is not usually regarded as a significantly porous medium.

Of the geophysical observations potentially useful for constraining crustal hydraulic properties at the required scale, seismic attenuation (total Q_t^{-1}) data in the crustal frequency band [0.08 — 25 Hz] offers definite advantages, if a causative mechanism related to fluid-rock interactions can be defined. Two main reasons justify investigating a relationship between crustal Q_t^{-1} measurements and crustal fluids. Intrinsic attenuation (dissipation), as embodied in the total loss factor Q_t^{-1} , is a measurement integrated over the seismically-sampled rock volume and typically covers up to the thickness of the crust and up to lateral scales of tectonic units. The ultimate transformation of seismic energy into heat is necessarily a function of the state of fluid and solid phases in the crust and of the hydraulic properties regulating the dissipation process.

The perception of a relationship between dissipation and pore-fluids goes back to the laboratory work of Born (1941). A relationship has since been demonstrated by numerous investigations of saturated rock samples. The Rock Physics Group at Stanford University, in particular, has identified a predominant dissipative mechanism — the "squirt-flow" process — common to all rocks tested and arising from localised, oscillatory viscous flow of thin-films of fluids trapped in the narrowest cracks of rocks with a distribution of pore aspect-ratios (for a summary, see Jones, 1986). The intrinsic Q_t^{-1} signature of this mechanism is a strong, bell-shaped frequency dependence in the kHz range. Its peak values are at well-

defined characteristic frequencies which depend on rock deformability, narrowest pore aspect-ratios, and fluid viscosity and temperature. At full saturation, the mechanism is most effective when the rock is sinusoidally sheared. This much is established for rock-samples but the applicability of the result to *in-situ* conditions is unclear, especially since the peak frequencies of the mechanism exceeds by a factor of 1000 the seismic peak frequencies.

Q_t^{-1} measurements for the lithosphere also show strong, bell-shaped frequency dependence but in the band 0.05 — 25 Hz, with conjectured peaks near one Hz. The only difference between tectonically active and passive regions is a lesser magnitude and frequency dependence for the latter (Aki, 1980; Sato, 1984). The lithospheric Q_t^{-1} signature, which is commonly attributed to attenuation in shear, shares similarities with that of squirt-flow. This suggests a link despite the difference in their respective observational bandwidth.

Current interpretations of Q_t^{-1} field-data in the lithosphere are heavily biased towards an elastic scattering process but they are not unique and little work has been done of late to search for a causative intrinsic mechanism that may be complementary to scattering. Inasmuch as the crust is interstitially hydrated, with pore-space existing at great depths, a dissipative frequency-selective mechanism analogous to squirt-flow may be operative in the crustal-seismic band. Although a simple version of squirt-flow has been modelled for crustal conditions by O'Connell & Budiansky (1977), their formulation of the physics of dissipation does not discriminate the more from the less dissipative cracks; it produces effectively frequency-independent dissipation. Thus, the conditions under which a highly frequency-selective squirt-flow analogue can operate in the crust remain unknown. Equally uncertain are the conditions under which such a mechanism can produce a significant signature on the measured attenuation at the surface of the Earth.

I argue here, using reported field measurements of the hydraulic and porous properties of crustal materials, that dissipation by localised viscous flow of thin films of fluid is likely to be significant in the crustal-seismic band as long as the locus of the mechanism is within the rough interspace of natural fractures rather than in rock microcracks. In the next section I present a compilation of crustal Q_t^{-1} data for two continental structures and critically examine the problems associated with current methods of interpreting crustal attenuation. Then I offer some elements of solution for a more developed interpretation. Conditions conducive to a seismic dissipation mechanism by viscous flow of crustal fluids are then elaborated, on the basis of an estimated characteristic frequency of squirt-flow for an hydrated crust whose fracture porosity exhibits characteristic dimensions.

2.1 Q_t^{-1} interpretation problem

The only accessible seismic data for inferring seismic attenuation in the Earth's interior are Q_t^{-1} measurements at its surface. In practice, sampling is pointwise in both frequency and location, and is assumed representative of a given geological environment by an appropriate combi-

¹. An earlier version of this paper was presented at the 4th annual meeting of l'Association canadienne-française pour l'avancement des sciences (ACFAS) in Edmonton, Alberta, on March 5th, 1993 (communication #12, Rouleau, P., H₂O: à quelle profondeur?)

nation of discrete measurements. As such, the problem of mapping attenuation from the surface data is inherently non-unique; however, inasmuch as Q_t^{-1} shows peculiar frequency dependence, the non-uniqueness can be reduced to an acceptable level by ancillary geophysical and geological constraints. Currently, the most plausible explanations for the frequency-dependent Q_t^{-1} observed in the lithosphere are: 1) elastic scattering by material heterogeneities in a single medium (e. g. Sato, 1982); 2) a depth-dependent combination of both scattering and intrinsic attenuation (e. g. Toksöz, Mandal & Dainty, 1990); and 3) an as yet undefined intrinsic attenuation mechanism (Aki, 1984).

Uniqueness remains unattainable because scattering and dissipation cannot be unequivocally separated from the single Q_t^{-1} quantity. That this is so for an heterogeneous medium such as the crust has been demonstrated by Wennerberg & Frankel (1989) who pointed out that generalised expressions for both intrinsic (dissipative) and scattering (non-dissipative) attenuation are isomorphic, despite the fundamentally different physics of the two phenomena. Thus, separation of these two equally plausible mechanisms can only be achieved by either *a priori* assumptions or independent information about the nature of material heterogeneities. The problem can be kept tractable by treating intrinsic and scattering attenuation as additive (Lerche & Menke 1986). This allows setting initial constraints, particularly with respect to the relative frequency dependence of the contributing mechanisms.

Since the seminal work of Aki (1980), interpretation of the observed frequency-dependent lithospheric $Q_t^{-1}(f)$ in the range [0.05 Hz < f < 25 Hz] has mainly been approached with the assumption that the frequency-selective mechanism of wave-amplitude decay is elastic scattering of primary seismic waves by large scale material heterogeneities. Several scattering theories are now available, some including intrinsic Q_i^{-1} (Wu & Aki, 1985; Frankel & Wennerberg, 1987; Zheng, 1991). Only a few successfully explain the ensemble of observations. The practice is to concentrate on the well-documented high-frequency side of the observational bandwidth where Q_t^{-1} is estimated from coda amplitudes (for a comprehensive review, see Herraiz & Espinosa, 1987).

Sato (1982), in particular, has been successful in fitting Aki's (1980) compilation over the entire band 0.05 — 25 Hz by using a scattering theory based on a mean-wave formalism, in which the lithosphere is treated as a continuous heterogeneous medium characterised by an isotropic distribution of fractional velocity fluctuations which singly back-scatter shear wave-fronts outside a cone of fixed apex angle.

In Sato's (1982) theory, attenuation is related to a scattering coefficient whose magnitude is a function of the mean value of the assumed velocity distribution and the corresponding spatial correlation function. If k_0 is a wavenumber [$k_0 = 2\pi/\lambda$; λ wavelength], ϵ^2 mean-square fractional velocity fluctuation, 'a' correlation length of scatterers, $\Gamma(v)$ gamma function, and v is the order of von Karman's autocorrelation function, taken as characteristic of the random velocity fluctuations (the autocorrelation

function relates ϵ^2 to 'a', with 'a' the dominant size of inhomogeneity) scattering attenuation

$$Q_{sc}^{-1}(k_0) = \frac{\epsilon^2 2\sqrt{\pi}\Gamma(v + \frac{3}{2})ak_0}{\Gamma(v)} \times \left[\frac{-1}{(\frac{1}{2} + v)} Z^{-(\frac{1}{2} + v)} \right] \Bigg|_{1 + \frac{a^2 k_0^2}{4}}^{1 + 4a^2 k_0^2} + \frac{\epsilon^2 \sqrt{\pi}\Gamma(v + \frac{3}{2})}{8\Gamma(v)(ak_0)^3} \times \left[\frac{1}{(\frac{3}{2} - v)} Z^{\frac{3}{2} - v} + 2\ln Z - \frac{1}{(\frac{1}{2} + v)} Z^{-(\frac{1}{2} + v)} \right] \Bigg|_1^{1 + \frac{a^2 k_0^2}{4}} \quad [2.1]$$

Z is a dummy variable evaluated at the bounds shown. k_0 can be converted to frequency f by using the relation $\lambda = V/f$, with V the mean velocity of shear-waves propagating in the heterogeneous medium.²

Eq-[2.1] contains two key elements: the frequency f_p at which scattering attenuation peaks, and the asymmetric sloping of $Q_{sc}^{-1}(f)$. The slope is positive for $f < f_p$ and negative for $f > f_p$. Both f_p and Q_{sc}^{-1} slopes depend on the size and distribution of heterogeneities relative to signal wavelengths. Eq-[2.1] was used by Sato (1982) to fit Aki's (1980) Q_t^{-1} data which cover only partially the observational bandwidth 0.05 — 25 Hz, with more data from tectonically complex regions near subduction zones and triple-junctions. Although the latter bandwidth is well documented by coda-wave attenuation measurements above one Hz and by surface-wave attenuation data round 0.05 Hz, there were no data in the decade 0.06 — 1 Hz (hence the dubbing "Aki's conjecture"). Because the displacement eigenfunction associated with a surface-wave of period 20 s ($f = 0.05$ Hz) extends beyond nominal crustal thickness and because scattering is likely from upper-mantle heterogeneities in tectonically complex areas where deep earthquakes occur, Aki's data set reflects properties of the "lithosphere" of which the crust is only a part.

In order to focus the discussion on crustal rather than lithospheric attenuation, I compiled crustal Q_t^{-1} data proper for the Eastern United States (EUS), a tectonically passive area, and for the Basin and Range province (BR), a tectonically active area, and plotted them versus frequency as Aki (1980) did. Both areas are well within the

²Eq-[2.1] differs from Sato's (1982) Eq-21 which does not have a logarithmic term; however, reconstructing his derivation, it is clear that his equation was misprinted and that Eq-[2.1] holds. Sato's formula is strictly valid for acoustic (scalar) scattering; he later developed a relation for elastic (vectorial) scattering using Born's approximation (Sato, 1984) but its implication does not affect the discussion presented here.

North-America plate and so presumably less tectonically intricate than subduction or triple-junction zones. This suggests much less scattering from the upper mantle. The result, spanning the bandwidth 0.08 — 20 Hz, is shown in figure-2.1 and 2.2 for EUS and BR, respectively, with the sources of data indicated therein. The band 0.06 — 1 Hz is well documented for these two areas. The wave types from which Q_t^{-1} was estimated include short-period fundamental and higher-mode Rayleigh waves, L_g waves and

Best-fit parameters are shown in Table 2. 1:, together with those found by Sato (1982) for Aki's data. Whereas Sato (1982) obtained a best fit for correlation length (or inhomogeneity size) $a = 4$ km, I get $a = 10.0$ km for EUS and $a = 8.4$ km for BR. Thus, typical inhomogeneity sizes are more than doubled for these continental crust environments. Although apparently successful, these modelling results help illustrate the difficulties of interpreting Q_t^{-1} in terms of scattering.

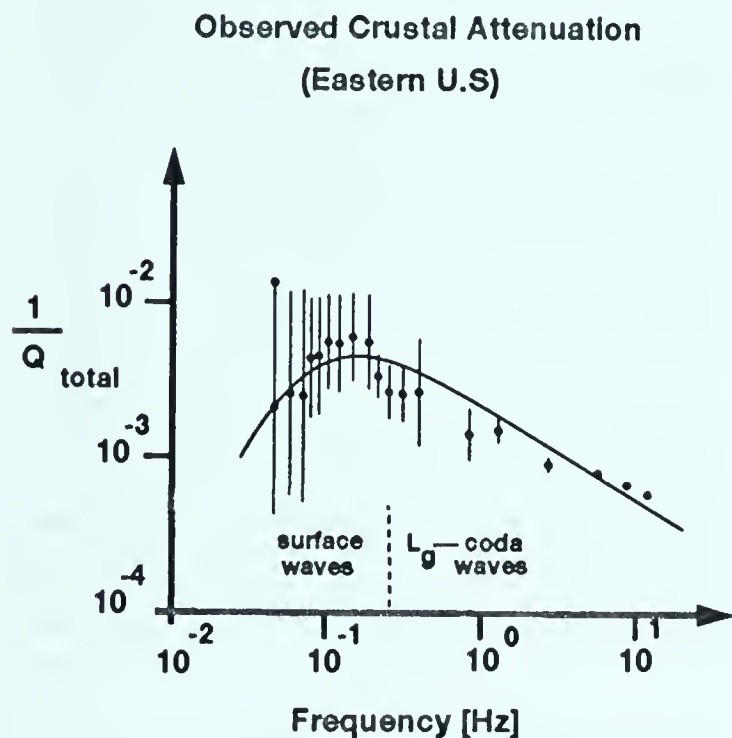


Figure-2.1 Compilation of crustal attenuation estimates for the Eastern United States Tectonic Province obtained from fundamental and higher-modes Rayleigh waves and from L_g waves and codas. Low-frequency data from Lee & Solomon (1975) and Mitchell (1981); high-frequency data from Singh & Herrmann (1983) and Mitchell (1980). Data fitting line from Eq-2.1 and Table 2.

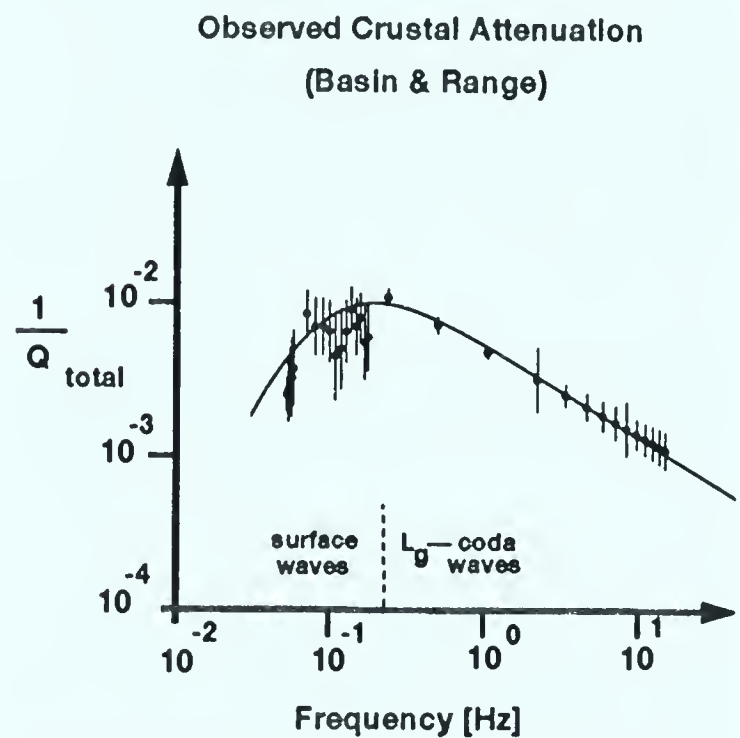


Figure-2.2 Compilation of crustal attenuation estimates for the Basin & Range Tectonic Province obtained from fundamental and higher-modes Rayleigh waves and from L_g waves and codas. Low-frequency data from Lee & Solomon (1975) and Patton & Taylor (1984); high-frequency data from Singh & Herrmann (1983) and Chávez & Priestley (1986). Data fitting line from Eq-2.1 and Table 2.

Table 2. 1: Best-fit parameters to crustal-attenuation model by scattering

Q_t^{-1} data	von Karman order (ν)	mean-square fractional velocity fluctuation (ϵ^2)	Correlation length of scatterers (a) [km]	mean velocity of shear-wave (V) [km/s]	Maximal attenuation frequency (f_p) [Hz]
EUS (fig-2.1)	1/3	0.003	10.0	3.8	0.1
BR (fig-2.2)	1/3	0.008	8.4	3.5	0.2
Sato (1982)	1/3	0.005	4.0	4.0	0.5

codas, all of which represent seismic energy mainly propagating in the crustal waveguide and generated by earthquakes well within the crust.

As for lithospheric Q_t^{-1} data, crustal Q_t^{-1} is smaller and less frequency-dependent in the passive EUS than in the active BR. I also fitted both sets of observations using Eq-[2.1] (smoothing lines in Figure-2.1 and Figure-2.2).

In Sato's (1982) approach $Q_t^{-1} = Q_{sc}^{-1}$ and stratification and intrinsic attenuation are disregarded. Material stratification, quite often characteristic of crustal structure at the tens of km scale, cannot be accounted for in the stochastic approach implicit in all wave-scattering theories³. Most

³ Except one-dimensional models in which the layering itself is random (Richards & Menke, 1983)

available scattering models assume weak scattering in order to invoke Born's approximation, despite its violation of the conservation of energy principle (the energy lost from the primary wave during scattering is neglected). This leads to logical inconsistencies in relating the amount of scattering (single or multiple) to the nature of scattering (backward or forward). These and other shortcomings have been discussed by Frankel and Wennerberg (1987) among others. A scattering theory which includes both multiple scattering and intrinsic attenuation (Wu & Aki, 1985) still relies on Born's approximation.

Conflicting results arise when different randomness models are used for characterising the heterogeneous crust. Gaussian and exponential autocorrelation functions yield a frequency-dependent Q_{sc}^{-1} which may be too small to be observed (Jannaud, Adler & Jacquin, 1991), whereas self-similar autocorrelation functions yield Q_{sc}^{-1} constant over wide frequency bands (Frankel & Clayton, 1986). These considerations cast doubts about interpreting Q_t^{-1} observations with too strong a bias towards a causative scattering mechanism.

In view of the purpose of constraining crustal hydraulic properties from Q_t^{-1} observations this is not a negative result since a contributory intrinsic mechanism, if found significant, carries much more information about rock-fluid interactions than provided by a scattering mechanism from which only impedance contrasts, vaguely related to fluids, can be inferred.

2.1.1 Intrinsic Q^{-1} interpretation

The classic approach to the attenuation problem is based on the modal theory of surface-waves (Anderson & Archambeau 1964), in which intrinsic causes are assumed and the Earth is regarded as a stack of laterally homogeneous layers. In this theory Q_t^{-1} is a weighted sum of intrinsic Q_i^{-1} and

$$Q_t^{-1}(\omega) = \int_{\text{surface}}^{\text{depth}} w(\omega, z) Q_i^{-1}(\omega, z) dz \quad [2.2]$$

where ω is angular frequency, z depth, Q_i^{-1} intrinsic attenuation, and $w(\omega, z)$ a weight function reflecting the relative density of the elastic energy stored in each layer. Eq-[2.2] shows that $Q_t^{-1}(\omega)$ is a function of the frequency dependence of both $w(\omega, z)$ and $Q_i^{-1}(\omega, z)$, themselves function of depth.

Eq-[2.2] provides a framework for mapping intrinsic attenuation and leads to an interpretation guided by pressure and temperature conditions in the Earth's interior. Early applications of the theory assumed frequency-independent Q_i^{-1} depth-distributions, (this allows linear inversion of the data) but later work (Jackson, 1971; Anderson & Given, 1982) introduced frequency dependence which was found necessary to account for a large class of attenuation measurements in the Earth. For example, Jackson (1971) explored the parameter space of frequency-independent attenuation depth-distributions by

applying Backus & Gilbert's (1968) inversion technique to mantle Love-wave Q_t^{-1} data characterised by a peak near 10^{-2} Hz. He concluded that no such model was satisfactory and then found, by a forward calculation, one model with a layer of highly frequency-dependent attenuation (the low-velocity zone) which did fit the data. Jackson stressed the importance of broadband data (at least two decades in frequency) in order to define the signature of an intrinsic mechanism characterised by a strong, bell-shaped frequency dependence.

The above method has also been applied extensively to the crust and upper mantle by Mitchell and colleagues who used fundamental and higher-mode Rayleigh wave observations from several tectonic provinces throughout the world, including EUS and BR (cf. Cong & Mitchell, 1988). Both fundamental and higher-mode data have been found necessary in order to reduce the trade-off between the effects on $Q_t^{-1}(\omega)$ of the depth-distribution and frequency dependence of $w(\omega, z)$ and $Q_i^{-1}(\omega, z)$ in Eq-2.2. The main conclusion of their studies is that dissipation within the lithosphere is significantly frequency-dependent in stable regions and much less so in active ones. This conclusion, however, is restricted in that the data are mainly confined to the band $10^{-2} - 1$ Hz, leaving the $1 - 10$ Hz decade unaccounted for. Q_t^{-1} is forced to be proportional to $\omega^{-\eta}$ with $\eta > 0$, which rejects the possibility of a bell-shaped attenuation.

Alternative approaches to the crustal attenuation problem consist in generating complete theoretical seismograms from an assumed source function and complex velocity distribution in order to extract Q_t^{-1} for comparison with observation (Hough, 1987; Toksöz et al., 1990). Such synthesis allows *ad hoc* incorporation of scattering attenuation which is disregarded in the more classical approach. It is important to emphasise that such syntheses have, so far, been performed only for limited portions of the crustal seismic bandwidth. Hough (1987) applied this technique to the crust underneath the ANZA broadband seismometer array (southern California) with the view that $Q_t^{-1}(\omega)$ is the sum of a frequency-independent part and a frequency-dependent part proportional to ω^{-1} . Her work, however, is essentially data fitting in which several parameters associated with the computed total attenuation have no physical basis whatsoever. Moreover, her result is limited to the sub-band $1 - 60$ Hz.

An approach similar to Hough's was employed by Toksöz et al. (1990) for studying crustal attenuation in Scandinavia. They took $Q_t^{-1}(\omega)$ as the sum of a scattering part, $Q_{sc}^{-1}(\omega)$; and two intrinsic parts, one frequency-independent, the other proportional to $\omega^{-\eta}$ with $\eta > 0$. All parts were allowed to depend on depth. The Q_t^{-1} data were fitted by trial-and-error, in the band $0.8 - 4$ Hz. The best-fit attenuation depth-distribution was found to depend on frequency accordingly to power $\eta = 0.5$ in the uppermost 2 km and to $\eta = 1$ down to 16 km, and frequency-independent beyond. Toksöz et al.'s interpretation of these results (which are non-unique) is that the ω^{-1} dependency reflects scattering, as suggested by some

scattering theories, and that the $\omega^{-0.5}$ dependency is due to fluid-flow losses in shallow fractures.

The reference to attenuation by fluid-flow losses originates from an earlier contribution (Toksöz et al., 1987) in which Biot's theory of wave propagation in permeable media (Biot, 1956) was invoked in order to interpret crustal $Q_i^{-1}(\omega)$ data from Maine, after they were corrected for scattering using Wu & Aki's (1985) theory. Observing that the remaining attenuation at high frequencies (> 1 Hz) was similar to that beyond a critical frequency predicted by Biot and which depends on the medium's average permeability, they surmised that their data could constrain the permeability of upper crustal layers.

This critical frequency separates the attenuation response accordingly to the predominance of viscous losses at low frequencies and of inertial losses at high frequencies, the latter process requiring the presence of large pores in the permeable structure. However, for Biot's critical frequency to fall near one Hz, the required permeability (~ 300 Darcy) exceeds by several orders of magnitude that inferred from many reservoir-induced seismicity observations which are reasonable estimates of crustal permeabilities (Brace, 1984; Talwani & Acree, 1985). Therefore, Toksöz et al.'s (1987, 1990) conclusion in this regard is questionable.

2.2 Elements of solution

The apparent impasse in consistently interpreting crustal Q_i^{-1} data can be summarised as follows. Interpretations solely based on a scattering mechanism do not account for stratification of the crust and lead to ambiguous results with respect to the assumed spatial distribution of heterogeneities. Interpretations based on intrinsic causes, whether accounting for scattering or not, are based on methods which have not fully explored the parameter space of frequency-dependent dissipation mechanisms possibly operating in the crustal-seismic band. Dissipation observed in saturated rock-samples, although exhibiting a bell-shaped $Q_i^{-1}(\omega)$ function similar to that reported for field Q_i^{-1} measurements, peaks at kHz frequencies with tails far removed from the crustal-seismic band.

I present below some basic elements for a solution to this impasse by first considering a single medium in which both intrinsic and scattering attenuation occur, and then considering the effects of stratification in such a medium.

2.2.1 Approach to a solution for a single medium

The first step is to assume

$$Q_i^{-1}(\omega) = Q_i^{-1}(\omega) + Q_{sc}^{-1}(\omega) \quad [2.3]$$

This relationship was found theoretically valid up to first order in fractional heterogeneity magnitude by Lerche & Menke (1986) for stratified media having statistical properties (density, shear and compressional velocity) which

are related by a scaling law, and in which intrinsic attenuation predominantly occurs in shear. These latter restrictions are reasonable for crustal conditions.

The second step is to find quantifiable representations for $Q_i^{-1}(\omega)$ and $Q_{sc}^{-1}(\omega)$. This purpose is served by using the isomorphic expression developed by Wennerberg & Frankel (1989) within the framework of linear anelasticity theory for intrinsic attenuation and of mean-wave theory for scattering attenuation. Either type of attenuation can be written as:

$$Q^{-1}(\omega) = M \int_{-\infty}^{\infty} D(\tau) \frac{\omega\tau}{1 + \omega^2\tau^2} d\tau \quad [2.4]$$

where M , $D(\tau)$, and τ have different physical interpretations depending on the causative mechanism. For intrinsic attenuation, M is the overall fractional elastic defect, τ the relaxation time associated with the lag of strain behind stress for a single Zener ($\equiv$ standard-linear) solid, and $D(\tau)$ a distribution over time scales of all Zener components contributing to the elastic defect. For scattering attenuation, M is the variance of fluctuations in velocity, τ the time delay associated with a single scatterer of finite size, and $D(\tau)$ a distribution of all scatterer sizes contributing to the attenuation.

Application of Eq-[2.4] to both $Q_i^{-1}(\omega)$ and $Q_{sc}^{-1}(\omega)$ requires further restrictions. To this end, I shall explore here the possibility that the frequency dependence of total $Q_i^{-1}(\omega)$ in Eq-[2.3] is dominated by intrinsic causes, and exploit the concept of self-similarity as it applies to anelasticity and mean-wave theory. In these two contexts, self-similarity is incorporated via the distribution function $D(\tau)$ which can be specified so that each elementary component implicit in Eq-[2.4] contributes the same amount of attenuation within a τ range of finite extent. Physically, this corresponds to a continuous, linear superposition of all contributing mechanisms, whose scale invariance is bounded. The distribution $D(\tau)=1/\tau$ for $\tau_1 \leq \tau \leq \tau_2$ and zero otherwise implements this in the anelasticity (Liu, Anderson & Kanamori 1976) and mean-wave (Wennerberg & Frankel 1989) contexts. The choice of τ_1 and τ_2 is, of course, critical.

With such a distribution, Eq-[2.4] integrates to

$$Q_x^{-1}(\omega) = \frac{M}{Ln(\tau_2/\tau_1)} \text{ArcTan} \left[\frac{\omega(\tau_2 - \tau_1)}{1 + \omega^2\tau_1\tau_2} \right] \quad [2.5]$$

where $x = i$ or sc . For a large difference ($\tau_2 - \tau_1$), $Q_x^{-1}(\omega)$ is constant within a correspondingly large frequency band; whereas for a small difference, $Q_x^{-1}(\omega)$ is portrayed by a narrow, bell-shaped function. Eq-2.3 and Eq-2.5 allow one to quantify the basic conditions under which a frequency-selective intrinsic mechanism can dominate the total attenuation in a single medium which both dissipates and scatters seismic energy. These conditions can then be evaluated for consistency with geological and geophysical constraints on the hydraulic and heterogeneous nature of the crust.

I calculated $Q_i^{-1}(\omega)$ for $0.01 \text{ Hz} \leq f \leq 100 \text{ Hz}$ by impos-

ing a peak value of $1/100$ at frequency 0.2 Hz in order to roughly approximate the attenuation observed in the crust. The result, shown in figure-2.3, is a good fit to the data from BR. The $Q_t^{-1}(\omega)$ curve is obtained for an intrinsic $Q_i^{-1}(\omega)$ which is self-similar within the narrow range $0.1 \leq f \leq 0.2$ Hz (maximal $Q_i^{-1} = 1/120$), and for a self-similar scattering $Q_{sc}^{-1}(\omega)$ within the much larger range $0.5 \text{ Hz} \leq f \leq 3 \text{ Hz}$ (maximal $Q_{sc}^{-1} = 1/400$). In terms of wavelengths (assuming $V \sim 3.5$ km/s), the self-similar scattering Q_{sc}^{-1} corresponds to heterogeneity sizes ranging $1 - 7$ km, as often inferred from P-wave travel-time fluctuation studies based on dense seismometer arrays (e. g. Flatté & Wu, 1988). The attenuation value for the plateau part of $Q_{sc}^{-1}(\omega)$ is inspired from Frankel & Clayton's (1986) results for a self-similar scattering medium characterised by a constant power spectrum of spatial velocity fluctuations for $k_0 a > 1$, for which the corresponding attenuation is frequency-independent. The value chosen for the plateau ($1/400$) is smaller than those calculated by Frankel & Clayton (1986); however, their method of estimating attenuation magnitude has been shown to overestimate by a factor of ~ 2 (Jannaud et al., 1991).

For the part of $Q_t^{-1}(\omega)$ contributed by $Q_i^{-1}(\omega)$, a peak $Q_i^{-1} = 1/120$ is compatible with the BR crust (Patton & Taylor, 1984). The length-scale L associated with the restricted range $0.1 \leq f \leq 0.2$ Hz can only be estimated from a specified dissipation mechanism. For squirt-flow, the characteristic frequency f_c at which dissipation is maximal is given by (Jones, 1986)

$$f_c = \frac{B}{\mu} \left(\frac{h}{L} \right)^3 \quad [2.6]$$

where B is effective rock bulk-modulus, μ fluid viscosity, and h the thickness of a typical dissipative crack of very low aspect-ratio h/L . For an hydrated crust, $B \sim 40$ GPa (granite) and $\mu \sim 10^{-4}$ Pa-s (liquid-water at 300°C and 100 MPa) are representative, which imposes an aspect-ratio $h/L \sim 10^{-5}$ to obtain $f_c \sim 0.2$ Hz. Setting $h \sim 10^{-8}$ m gives $L \sim$ one mm, both justifiable estimates as discussed in section-2.3.

The approach described above supposes that intrinsic attenuation dominates scattering in the observed $Q_t^{-1}(\omega)$. This is an uncommon assumption requiring justification for each portion of the wave field contributing to the crustal-seismic band. For the low-frequency part documented by fundamental and first higher-modes surface-waves it can be argued (Mitchell and colleagues) that the corresponding wavelengths are much larger than expected heterogeneity dimensions and so Q_t^{-1} from these waves mainly reflects intrinsic attenuation. For the high frequency portion where scattering effects are most likely to be manifest, justifying the predominance of dissipation is not straightforward and requires detailed discussion, especially with regards to coda-waves generated by local earthquakes (within ~ 100 km) from which many measurements are made.

Composite Attenuation Model for Self-similar, Anelastic & Heterogeneous medium

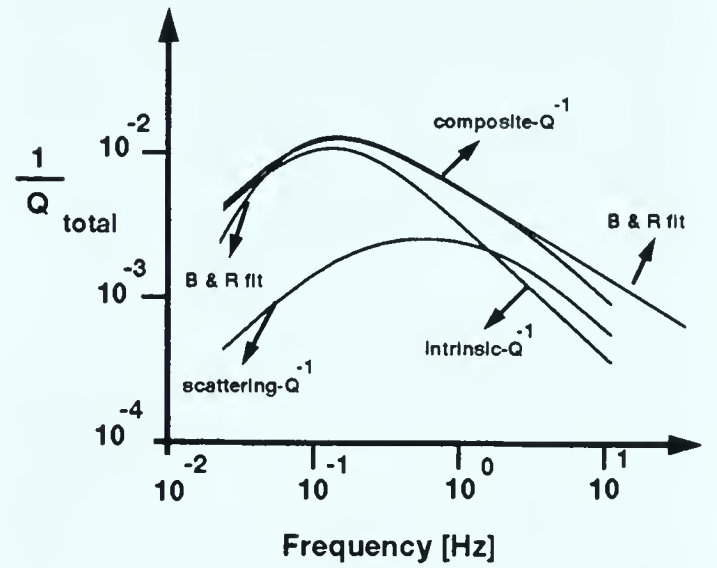


Figure-2.3 Total attenuation for a single medium in which seismic waves are attenuated by the sum of self-similar intrinsic and scattering mechanisms which span narrow and wide frequency bands, respectively. Associated length-scales are $\sim$ one mm for dissipation, $1 - 7$ km for scattering. The composite attenuation (sum) overlaps data fit for Basin & Range (fig-2.2), except at very low and very high frequencies. Composite attenuation peak at 0.2 Hz.

Q_t^{-1} from local coda-waves is usually inferred from the viewpoint that the decaying envelope of the coda reflects both scattering and dissipative attenuation, the exact proportion of each being determined from a specific theoretical framework. For frameworks based on scattering and built using Bom's approximation, the proportion due to each cause varies widely and depends on the shape of the seismic energy-density curve versus distance from the source (e. g. Wu & Aki, 1985). However, for frameworks based on conservation of energy, scattered waves, while producing a major effect on the onset of the coda, have little influence on the decaying envelope, which then predominantly reflects intrinsic attenuation (Frankel & Wennerberg, 1987; Wennerberg, 1993). Thus, since nearly all reported coda- Q^{-1} values are estimated from time windows well within the decaying envelope, there is good reason to view them mainly as a measure of intrinsic attenuation. It may be argued that this result is limited to local data; however, similarities between reported local coda- Q^{-1} values and the long-range (~ 1000 km) L_g -wave Q^{-1} -values (Nuttli, 1988) justify extending this reasoning to much of the crust.

2.2.2 Approach to a solution for a layered medium

An estimate of the total attenuation at the surface of a stratified heterogeneous and anelastic medium requires ways of stacking $Q_t^{-1}(\omega)$ curves of the type shown in fig-

ure-2.3. Each curve represents dissipation and scattering at depth z of a discrete-layer model. For this purpose, the theory of surface-wave attenuation of Anderson & Archambeau (1964), although limited to flat layers in which only intrinsic mechanisms operate, is useful. It ensures continuity of displacement and stress at contacting layers and so of attenuation. The approach described below is given in terms of Love-waves which suffices to illustrate the main features, but it can be easily extended to Rayleigh-waves which are necessary in order to synthesise much of the wave field from which crustal Q_t^{-1} is estimated [for example, by superposing many higher-modes to reproduce L_g wavetrains, (Kennett 1986)].

For Love-waves, attenuation at the surface of $n-1$ discrete layers overlying a half-space at level n can be expressed by a specialised form of Eq-2.2 due to Knopoff (1964), which reads:

$$Q_t^{-1}(\omega) = \sum_{j=1}^n \frac{\beta_j}{c} \frac{\partial c}{\partial \beta_j} [Q_i^{-1}]_j \quad [2.7]$$

where j is a layer index, c surface-wave phase velocity at angular frequency ω , β_j shear-velocity in layer j , and Q_i^{-1} intrinsic attenuation in layer j . The factor with the partial derivative accounts for that part of the elastic energy-density being stored at level j . Given an Earth model, these partial derivatives can be calculated using energy integrals over depth. The integrands comprise eigenfunctions arising from continuity boundary conditions (Anderson & Harkrider, 1968). The shape of each eigenfunction portrays the depth-range at which anelastic properties are sampled at a given frequency (low frequencies sample deeper).

Knopoff (1964) assumed that Q_i^{-1} was frequency-independent in all layers; however, because the integrations implicit in Eq-2.7 are over depth, Q_i^{-1} can be taken frequency-dependent (Jackson, 1971; Lee & Solomon, 1978; Anderson & Given, 1982). Extending Eq-2.7 to frequency-dependent $Q_i^{-1}(\omega)$ and invoking the additivity assumption (Eq-2.3), attenuation at the surface of a stratified, heterogeneous and anelastic medium can be approximated by

$$Q_t^{-1}(\omega) = \sum_{j=1}^n \frac{\beta_j}{c} \frac{\partial c}{\partial \beta_j} [Q_i^{-1}(\omega) + Q_{sc}^{-1}(\omega)]_j \quad [2.8]$$

Eq-2.8 is a "rough" estimate of total Q_t^{-1} because the weighting factors only account for the vertically integrated energy being attenuated. If heterogeneities in the medium are such that scattering is strong, then energy scattered in many directions must be considered. In such case, Eq-2.8 breaks down because the weight function no longer represents independent eigenmodes which couple at heterogeneity boundaries (Maupin, 1988). Nevertheless, when intrinsic attenuation predominates in all layers and when the layered crustal waveguide is relatively smooth, Eq-2.8 can be regarded as a first approximation which at least accounts for some energy being scattered.

Although techniques for computing weight factors of a dissipative crust model for any frequency range exist

(Schwab et al., 1984), they are difficult to implement at high-frequencies where many higher-mode phases coalesce to form a variety of wavetrains. With regards to constraining crustal hydraulic properties, the required effort seems futile without a fully explicit dissipation mechanism. However, an analogue to the crustal problem can be built by using a reference Earth model for which the weight factors are known with the chosen structure appropriately sampled, just as the crust would be by the multi-mode surface-waves propagating in it. This allows investigating the severity of the trade-off between depth and frequency dependence of attenuation.

I computed $Q_t^{-1}(\omega)$ from Eq-2.8 for four extreme cases of composite Q^{-1} distributions [i-e $Q_i^{-1}(\omega, z) + Q_{sc}^{-1}(\omega, z)$], using a Shield model for which the weighting factors have been tabulated by Anderson & Harkrider (1968). I chose the top thirty layers of their reference structure (total thickness = 270 km) and the corresponding weighting factors for the fundamental mode of Love-waves in the period range 10 — 100 sec (to each period is associated thirty weights). This range covers only one decade in frequency and so provides a worst case scenario for discriminating the possible $Q_t^{-1}(\omega)$ signatures. Plots of weight-factors versus depth (not shown) reveal that the structure is evenly sampled by the fundamental modes. In analogy with resolvable crustal- Q^{-1} models, I divided the reference structure in three layers with thicknesses of 30 km, 100 km, and 140 km, respectively, and assumed a perfectly elastic underlying half-space (weight factors are very small beyond 270 km for the periods used). In terms of relative thickness in a 35 km-thick crust, these layers correspond to a 4 km cover, 13 km mid-layer, and an 18 km bottom. I also assumed composite Q^{-1} to be dominated by intrinsic attenuation and took scattering Q_{sc}^{-1} frequency-independent. The principle of the calculation is depicted in figure-2.4.

Figure-2.5 shows selected $Q_t^{-1}(\omega)$ signatures for the four limiting cases considered: 1) a frequency-independent composite attenuation which decreases with depth, accordingly to $Q_1^{-1} = 1/150$, $Q_2^{-1} = 1/300$, and $Q_3^{-1} = 1/900$; 2) a depth-distribution of a bell-shaped, frequency-dependent composite attenuation with peak values identical to those of the first case, with characteristic frequencies $f_{c1} = 1/60$ Hz, $f_{c2} = 1/40$ Hz, and $f_{c3} = 1/30$ Hz (schematised in figure-2.4); 3) a bell-shaped, frequency-dependent composite attenuation in the middle layer of peak value $Q_2^{-1} = 1/90$ at $f_{c2} = 1/50$ Hz, with a top layer of constant $Q_1^{-1} = 1/150$ and a bottom layer of constant $Q_3^{-1} = 1/900$; and 4) a frequency-independent composite attenuation as in case-1 but with a low- Q^{-1} mid-layer set at $Q_2^{-1} = 1/90$. The shift of peak attenuation values to higher frequencies as depth increases (cf. fig-2.4) was chosen to reflect the effect of increasing temperature (decreasing fluid viscosity) on the characteristic frequency of squirt-flow (Eq-2.6).

Frequency-dependent Composite Attenuation Model In a Stratified, Anelastic & Heterogeneous Structure

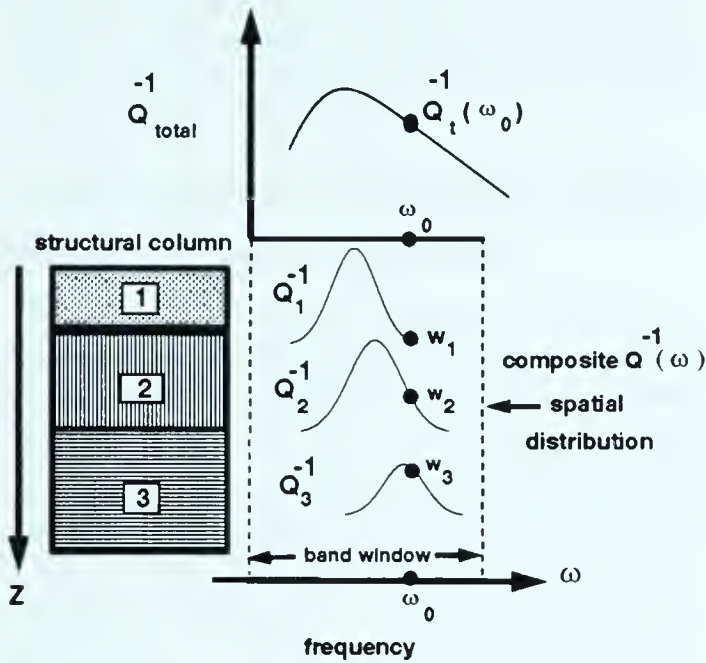


Figure-2.4 Crustal $Q_t^{-1}(\omega)$ analogue for a reference three-layer structure attenuating seismic waves by dissipation and scattering. Composite- $Q^{-1}(\omega)$ is the sum of scattering and intrinsic dissipation in each layer, the latter dominating. For each frequency ω_0 , Q_t^{-1} is a weighted average of the spatially distributed composite- Q^{-1} , with weighting factors w reflecting elastic energy-density stored in each stratum, in accordance with Eq-2.8.

Total Attenuation of Love-waves for Layered, Heterogeneous & Anelastic Structures

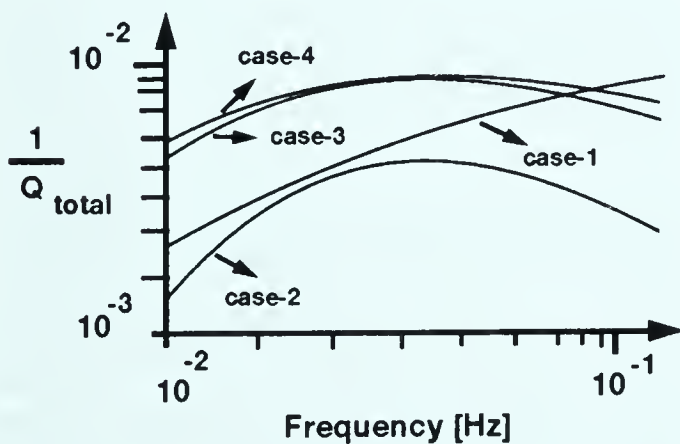


Figure-2.5 Total attenuation for a three-layer medium in which seismic waves are attenuated by the sum of self-similar intrinsic and scattering mechanisms (Eq-2.3 & 2.5). Calculations from Eq-2.8, using partial derivatives of Anderson & Harkrider (1968) for a Shield structure. Numbered cases refer to modelled profile of composite attenuation. Case-1: frequency-independent composite Q^{-1} diminishing with depth. Case-2: bell-shaped, frequency-dependent composite- Q^{-1} peaking to higher frequencies as depth increases (see fig-2.4). Case-3:

frequency-dependent composite- Q^{-1} in mid-layer only. Case-4: frequency-independent composite- Q^{-1} everywhere, with lowest attenuation in mid-layer.

Given that the computed $Q_t^{-1}(\omega)$ curves are plotted in log-log space, it is seen that there are major differences between all four composite Q^{-1} models, the largest discrepancy arising from the constant- Q^{-1} depth-distribution with no low- Q^{-1} zone (case-1), which yields a monotonically increasing $Q_t^{-1}(\omega)$ curve. All other curves exhibit frequency-dependence with identifiable maximal values. $Q_t^{-1}(\omega)$ for case-2, the bell-shaped Q^{-1} model shifting to high-frequencies with depth, peaks at $f_c \sim 1/28$ Hz where $Q^{-1} \sim 1/250$, thereby illustrating that the apparent characteristic frequency is not simply related to those operating within the structure. $Q_t^{-1}(\omega)$ curves for the model with the bell-shaped $Q^{-1}(\omega)$ in the mid-layer (case-3) and for the constant- Q^{-1} depth-distribution with low- Q^{-1} zone (case-4) show little discrepancies, except as frequency increases either side of their common peak values. Unlike case-2, the apparent characteristic frequency of case-3 ($\sim 1/40$ Hz) is much closer to that of the mid-layer $Q^{-1}(\omega)$ and so provides a rough estimate for the intrinsic value of f_c . Although the severity of the trade-off between depth and frequency-dependence of attenuation is well illustrated by cases 3 and 4, a wider bandwidth would allow resolution of the two composite $Q^{-1}(\omega, z)$ distributions. Since the observational bandwidth for the crust covers three frequency-decades, this is a reasonable expectation. Therefore, it can be concluded that an attenuation mechanism characterised by a bell-shaped $Q^{-1}(\omega)$ relation has an identifiable signature at the surface of a stratified medium such as the crust, provided the observational bandwidth is wide enough.

The above results are obtained from highly idealised conditions and, of course, are subject to confirmation by proper computations for realistic crustal attenuation models, although they are qualitatively similar to those formally obtained by Cong & Mitchell (1988). Eq-2.8 offers one approach to do so, in which both intrinsic and scattering attenuation models in stratified structures can be explored and compared with observation. The key assumption is the additivity of intrinsic and scattering attenuation, which has solid theoretical support for layered anelastic media with statistically distributed material impedances (Lerche & Menke, 1986). Because impedances are random in Lerche & Menke's theory, the concept of partial derivatives implicit in Eq-2.8 needs revision, preferably along the line of accounting for the effect on phase velocity c of a variation of β_j associated with its randomness. This should yield more realistic weighting factors for the attenuation part contributed by scattering and so permit proper examination of a large class of scattering- Q_{sc}^{-1} models.

With respect to realistic dissipation models, it is uncertain whether the phenomenological representation of Eq-2.5, with linear sloping about a characteristic frequency, is adequate. Asymmetric sloping is desirable if only to fit the skewness of the observed crustal $Q_t^{-1}(\omega)$ at low fre-

quencies (cf. fig-2.3). Although such sloping is predicted in the kHz band by the explicit squirt-flow model of Murphy, Winkler & Kleinberg (1986), their model is strictly valid for granular rock-samples and its applicability to crustal formations remains to be shown. The key parameter here is the characteristic frequency of the mechanism (Eq-2.6), and the conditions which yields a value well within the crustal-seismic band must first be identified.

2.3 Conditions for seismic dissipation by viscous fluid-flow

In principle, a wealth of dissipation mechanisms can operate when earth materials are sinusoidally stressed, as attested by numerous laboratory studies (for a review see Karato & Spetzler, 1990). Pertinent results can be summarised from the character of dissipation in frequency space which has been shown to differ accordingly to the saturation state of the rock. For dry rock-samples, dissipation has been observed effectively frequency-independent, except at upper-mantle temperatures where a mild dependency arises due to partial melting of the rock grains. For hydrated rock-samples at crustal conditions, dissipation is much larger than for the dry state and strongly frequency-dependent, with characteristic frequencies intimately related to the smallest pore-space available to free fluids in the distribution of pore aspect-ratios (Jones, 1986). This latter observation is remarkable, not only because it applies to all rocks tested, but also because it demonstrates the predominance of a single mechanism — the "squirt-flow" process — over all other possibilities.

The characteristic frequency of squirt-flow is given by Eq-2.6. In the laboratory, typical values fall in the kHz range, which constrains preferential aspect-ratios to $h/L \sim 10^{-3} - 10^{-4}$ for crustal fluids (Jones, 1986). For the mechanism to be operative in the crust, there must be characteristic aspect-ratios $h/L \sim 10^{-5} - 10^{-6}$, in sufficient amount so as to be manifest in the crustal- $Q_t^{-1}(\omega)$ signature when they are fluid-filled. *In situ* aspect-ratios are not amenable to direct observation; however, reliable inferences on h and L can be drawn from geophysical manifestations of the hydraulic nature of the crust, as fully discussed below.

Aki (1980) searched for a causative mechanism which could explain his Q_t^{-1} data, with conjectured peak near 0.5 Hz. He considered eight mechanisms by estimating their corresponding characteristic frequencies, of which he retained only two: scattering by km-scale impedances; and the thermoelastic process modelled by Savage (1966), in which dissipation results from irreversible heat flow at the grain/crack-scale, associated with adiabatic deformation of polymineralic cracked-rock. Savage's mechanism has characteristic frequency $f_c = \kappa/(0.5 L)^2$, where κ is grain thermal diffusivity and crack-length $L \sim$ one mm for $f_c \sim 0.5$ Hz. In itself, this one mm length-scale supports the narrow range of self-similar dissipation which I used in section-2.2 for characterising intrinsic attenuation; however, Savage's (1966) model is built on the premiss that cracks are well isolated from one another, a situation quite unlike typical rock formations

where at least some connectivity is known to exist.

The mechanisms rejected by Aki (1980) include squirt-flow for which he used an aspect-ratio of 10^{-3} ; but, this value only typifies cracks in rock-samples. Even though such cracks certainly occur in the crust, the squirt-flow dissipation contributed by them, clearly, cannot appear in the low-frequency crustal-seismic window. However, crustal formations are known to be fractured over a wide range of scales, giving rise to another level of pore geometry: fracture-porosity. Much work has been devoted of late to characterise this porosity and dimensional constraints provided by recent field observations have not been examined from the viewpoint of seismic dissipation. Thus, the conclusion that squirt-flow is irrelevant in the crustal-seismic band is subject to profound revision if the locus of dissipation is within fracture interspace. For the latter to be likely, crustal formations must, of course, be significantly hydrated.

2.3.1 Fluids in the crust

Evidence for the pervasiveness of H_2O -rich fluids throughout the Earth's crust is now abundant. In broad terms, geochemical budgets of metamorphic reactions of dehydration indicate that the overall water content circulated in the crust/upper mantle system amounts to 3 to 6% of the mass of the crust, i.e roughly the mass of the Earth's current oceans (Fyfe et al., 1978). The degree and mode of saturation vary with geological conditions, tending to be high and in connected fractures in hydrothermal regimes, low and in isolated porous systems elsewhere. Figure-2.6 shows a conceptual model of the continental crust, inspired from Gough (1986), which helps discuss the relevant information. Gough (1986) summarised the main geophysical features of the continental crust according to the character of seismic reflectors, electrical conductivity, state of stress, and crustal fluids. In his classification, the upper crust is brittle, seismically transparent and electrically resistive, whereas the lower crust is ductile, opaque and conductive in the same

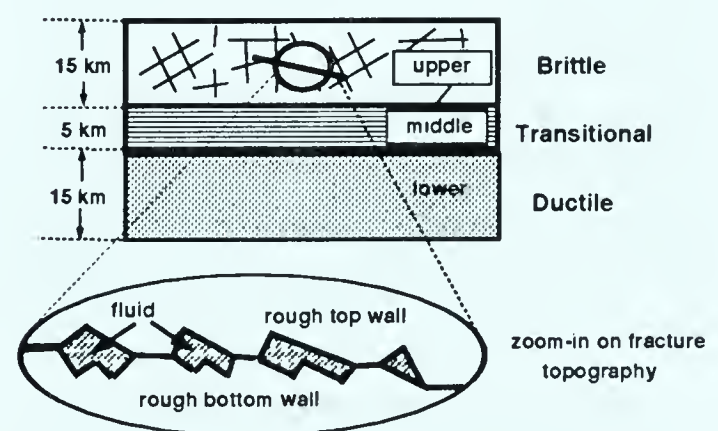


Figure-2.6 Conceptual model of an hydrated crust divided accordingly to rheological regime. Upper crust is brittle, with extensive fracture networks. Fracture walls are rough, providing crack-space available to free fluids. Middle crust is rheologically dual, with allowance for sub-horizontal fractured zones pressurised by episodically trapped fluids. Lower crust is ductile, with allowance for equilibrated pore-geometry in saturated laminar

formations. Fluid may be of both metamorphic and meteoric origin. Thicknesses are relative. Void topography of fracture interspace has characteristic dimensions.

respects. He suggested that observational constraints favour the presence of saline fluids throughout the crust but in two different modes: in the upper crust water resides in isolated cavities when stress is compressive (the commonest situation); and in the lower crust water resides as interconnected films on crystal surfaces and may be absent beneath shields. Departures from these generalisations are expected, particular at mid-crustal depths where brittle and ductile regimes likely join to form a transitional layer of finite thickness.

The presence of fluids in the upper crust has been confirmed by ultra-deep drilling in the Kola Peninsula, where 12 km of predominantly metamorphic rocks has been traversed in the billion-year old Baltic Shield (Kozlovsky, 1987). Trapped mineralised water, released by the drilling bit, was found profusely flowing from several km-thick fractured layers and down to 9 — 10 km, a result previously thought impossible. That free aqueous fluids are preferentially emplaced in upper-crustal fracture networks is an important factor to recognise in modelling seismic dissipation and other geophysical observables. It is worthwhile noting that the term "isolated cavities" used by Gough (1986) is unspecific as to dimensions; the Kola deep-drilling result clearly demonstrates that voids at the fracture-porosity level are rather well-connected. However, individual fracture networks, with nominal thickness of a few km, may be spatially disconnected from one another and so appear as poor conductors at a larger scale.

Less direct evidence for the occurrence of upper crustal fluids is provided by manifestations of induced seismicity often accompanying large-scale reservoir impoundments (Talwani & Acree, 1985). The depth of triggered events, many of which considerably lag impoundment, ranges from a few km down to 20 km over areas often exceeding reservoir length-scales. Although the exact process and conditions by which seismicity is induced are still poorly understood, water is the prime factor in favouring rupture via lubrication of fault walls. This reservoir-induced-seismicity phenomenon (RIS) also provides a method for estimating crustal permeability. Reported estimates from 22 RIS cases in vastly different geological settings yield the surprisingly narrow range 0.1 — 10 milliDarcy, with evidence that permeability decreases with depth, as expected (Talwani & Acree, 1985). That this range is narrow indicates that preferential fluid conduits, of dimensions common to many rock formations, dominate the overall fluid conductivity of crustal rocks. The measured permeabilities are orders of magnitude larger than that measured on laboratory-scale samples, a fact again indicating the predominance of fracture porosity over that of intact granular rocks.

Other geophysical evidence of fluid occurrence is indirect, particularly for deeper parts of the crust where it is inevitably circumstantial. Shear-wave records from three-component broadband seismometers installed in various geological settings generally show marked energy splitting into two waves, each with a statistically significant polarised direction (Crampin, 1987). The quasi-universality of such observations is coherently explained by the

extensive-dilatancy model (EDA), in which shear-waves are split as they are transmitted through a medium pervaded by fluid-filled cracks, many of which are aligned perpendicularly to the direction of the ambient minimum principal stress. Travel-time differences between the two split waves constrain the density of saturated cracks, parameterised by $\epsilon_c = N \langle r_c^3 \rangle$ with N the number of cracks of radius r_c per unit volume (brackets denote averaging). Observation typically gives $0.03 < \epsilon_c < 0.07$, the lower bound of which corresponds to a crack of radius 0.7 in a unit cube. The EDA interpretation rests on the assumption that wavelengths much exceed crack sizes; therefore crack dimensions cannot be resolved. The depth range of EDA occurrence is also uncertain; however, it is likely to be limited to the upper 20-km of the crust. The main result here, other than strongly suggesting the pervasiveness of crustal fluids, is the estimated crack density ϵ_c , which provides some dimensional constraints on at least those cracks that are stress-aligned.

Electrical resistivity observations from magnetotelluric depth-soundings, combined with collocated multi-channel seismic reflection surveys, have provided much insight into the petrological state of crustal-rock formations, especially when interpreted in terms of ionic properties of crustal fluids in pore-geometries conducive to localised large-scale conduction (Hyndman & Shearer, 1989; Marquis & Hyndman, 1992). Although there are exceptions, the resistivity depth-profile generally exhibits a sharp decrease beyond mid-crustal levels wherever temperature reaches 400 ± 50 °C. Depending on geologic age, upper-crustal resistivities ($z < 13 - 20$ km) range $10^2 - 10^5 \Omega \text{ m}$ (dry rock $\sim 10^4 \Omega \text{ m}$), whereas lower-crustal values, referenced to a thickness of 10 km, average $20 \Omega \text{ m} - 500 \Omega \text{ m}$ (sea water $\sim 10^{-2} \Omega \text{ m}$), with the lower bound characteristic of most Phanerozoic (younger) areas. For deep layers of low resistivity, a consistent explanation, albeit non-unique (cf. Frost et al., 1989), is found by invoking the presence of a few per cent free saline-water in pressurised, finely granular lamellae with effective pore-aspect ratios ranging 0.1 — .03, which then also explains observed reflection coefficients associated with seismic reflectors (Hyndman & Shearer, 1989). This model requires deep saturated formations to reside as such over long geological periods, which calls for complex mechanisms of retention despite buoyancy-related instabilities (e. g. Bailey, 1990). Although upper level resistivities are higher than below, they do not exclude fluid occurrence since field-estimates, either shallow or deep, often exceed by orders of magnitude laboratory-scale measurements, which again favours fracture porosity in the upper crust.

2.3.2 Quantitative constraints

It is transparent from the above summary that crustal fluids preferentially occur in the interstices of fractures, at least in the upper crust. It is quite logical then to expect that dissipation by viscous fluid-flow operates therein. This view invites focussing on the dimensions of these interstices in order to examine the operability of the squirt-flow mechanism in the crustal-seismic band, using its characteristic frequency — Eq-2.6. For lower-crustal

conditions, where a ductile regime dominates and so implies rapid fracture sealing, squirt-flow can still operate provided grains of the ductile rock-matrix respond nearly elastically to seismic stressing. Given the high frequencies of seismic waves relative to tectonic forcing the latter is likely. Physical conditions existing in the brittle/ductile transitional zone are most uncertain; therefore, dissipation within is difficult to characterise. This problem is made worse by poor control on the depth and thickness of the dual layer. At best, magnetotelluric soundings can only constrain the location of the resistivity change, not the thickness of least resistive formations. Seismic velocity depth profiles exhibiting low P-wave amplitude zones provide some clues, but their interpretation is non-unique (Benz, Smith & Mooney, 1990). What seems to be clear though is the demarcation set by the 400 ± 50 °C isotherm. Below this isotherm earthquakes rarely occur and this suggests that temperature limits pervasive fracture porosity.

Because crustal thermodynamic conditions apparently change at the 400 ± 50 °C isotherm, the squirt-flow characteristic frequency may change accordingly. If Hyndman & Shearer's (1989) stress-balanced, ductile pore-geometry model is accepted for 10 km-scale portions of the lower crust, then effective aspect-ratios of 0.1 — 0.03 cannot produce a characteristic frequency in the crustal-seismic band if the electrically-conducting fluid is saline water. Support for low attenuation in the deepest part of the lower crust comes from crustal Q_t^{-1} inversion models, which invariably attribute very low values at these depths (Cong & Mitchell, 1988). However, low-resistivity saturated layers may very well be confined to a narrow strip at mid-crustal levels within the brittle zone (Bailey, 1990), where seismic attenuation is often inferred to be high (e. g. Herrmann & Mitchell, 1975; Hough, 1987).

Perhaps the nature of the conducting fluid is not as simple as that of a single brine, as pointed-out by Hyndman & Shearer (1989). Moreover, implicit in their modelling of a ductile matrix is the premiss that under ambient pressure and temperature rock-grains assume a configuration which minimises surface-energy, idealised as faceted boundaries of perfect curvatures. However, some roughness is inevitable at the scale shared by molecular and macroscopic processes. It may be that dissipation occurs by rough grain/fluid boundary relaxation, with weak fluid bonds as the main absorbing agent. If so, and if the mechanism is akin to squirt-flow, then for its characteristic frequency to fall within the crustal-seismic band the viscosity of the molecular fluid must be of the order of 10^6 Pa-s. I do not know of any experimental result on bounded aqueous-fluids which would support such high viscosity, which is somewhat representative of some polymers. Work on electrically conductive crustal fluids is only commencing (Nesbitt, 1993), the emphasis being put on the effect of variations of pressure, temperature, and composition. The nature of the viscosity of such fluids as they adhere to moving solid-surfaces invites attention, preferably within the framework of physical chemistry of surfaces.

For upper-crustal conditions, the situation is much less intricate because fracture porosity can be reasonably characterised. Profiler measurements of the roughness of natural fracture surfaces, with the aim of understanding

fault instability, provide constraints on void dimensions (Brown & Scholz, 1985, 1986). That these fractures are rough is a most important factor (cf. fig-2.6 for a visualisation). Fracture surfaces are typically fractal as evidenced by their power-law distribution in the wavenumber domain; therefore, they do not have a characteristic dimension. However, when two such surfaces are joined and allowed to progressively close with burial, they do develop a characteristic length — the correlation length λ_c — as a result of sequential closure starting with the longest wavelength asperities (Scholz, 1988). These large asperities support excess burial stress, leaving void space available to great depths (e. g. Gavrilenko & Guégen, 1989).

Based on a simple effective pressure-depth relation, Scholz (1988) found λ_c to vary asymptotically from 10^{-1} m at $z \sim 1 - 2$ km to 10^{-3} m throughout the seismogenic zone ($6 < z < 20$ km). λ_c has two important meanings: it is the length of the smallest asperity, and the length of the maximum space available to fluid. It also scales with asperity height, giving a constant aspect ratio. This ratio is small for upper-crustal conditions, constraining the space to be very flat (crack-like). Beyond λ_c , the open space geometry is fractal; however, the macroscopic flow of fluid in response to a steady pressure-gradient across a portion of it can still be reasonably modelled assuming smooth parallel walls (Brown, 1987). That λ_c is $\sim$ one mm for a significant part of the crust appears to support the characteristic scale of seismic dissipation assumed in section-2.2.1. However, the lowest aspect-ratio reported from fracture roughness data and extrapolated to crustal pressure conditions is $\sim 10^{-4}$, giving too high a characteristic frequency for squirt-flow. Considering that the pressure-depth relation used by Scholz (1988) is for purely elastic closure of "clean" surfaces, the aspect-ratio characterising rough fractures can be expected to be even smaller. An independent estimate of fracture void thicknesses, based on permeability estimates derived from reservoir induced seismicity (RIS), demonstrates that this is likely.

The common procedure for estimating crustal permeability from RIS is to record the evolving seismicity relative to water-level variations. Event locations and time-delays behind excess fluid-pressure onset are then analysed using the diffusion equation from which crustal diffusivity is estimated for calculating permeability. This method was applied by Talwani & Acree (1985) to events extending to depths of 20 km. As mentioned earlier, the observed narrow range of values (0.1 — 10 mD) typifies fracture permeability. Permeability characterises the connected geometry of a porous structure and RIS estimates reflect the *in situ* nature of this geometry, which is known to be rough at the scale of fracture walls. Extensive numerical modelling of laminar fluid flow through rough-walled (fractal) fractures show that macroscopic flow properties are dominated by a characteristic pore — the "critical neck" — which represents the smallest aperture along the path of highest aperture through the fracture (Pyrak-Nolte, Cook & Nolte, 1988). Viewing rough-fracture interspace as a network of connected tubes of relatively large aperture interspersed by narrow cracks, this "smallest aperture" logically equals Scholz's (1988) crack

height associated with correlation length λ_c .

In order to estimate, albeit roughly, the *in situ* thickness of the "critical neck" I used RIS-derived permeabilities together with Walsh & Brace's (1984) equivalent porous medium concept, in which permeability is simply related to a "typical crack" thickness, which I take as that separating two parallel plates. The relation is

$$K = \frac{d_c^2}{12} \quad [2.9]$$

where K is permeability (in m^2) and d_c is characteristic-crack thickness, i.e. that which is assumed to prevail in a distribution of void dimensions. Using Talwani & Acree's permeability measurements and Eq-2.9 I get $10^{-8} \text{ m} < d_c < 10^{-7} \text{ m}$. Combined with $\lambda_c \sim 10^{-3} \text{ m}$ for the seismogenic zone, I obtain for a characteristic crack the aspect-ratio $d_c/\lambda_c \sim 10^{-5}$, which, inserted in Eq-2.6, gives $f_c \sim \text{one Hz}$; therefore, well within the crustal-seismic band. There is no logical inconsistencies between the aspect-ratio extrapolated by Scholz (1988) from clean surface fracture-roughness data and that obtained here when considering that voids of *in situ* fractures are likely coated by chemically precipitated materials as often occurs following rupture by stress-corrosive hydraulic fracturing (e. g. Brantley et al. 1990). A few layers of precipitates deposited on surfaces of a low aspect-ratio pore can greatly reduce its thickness, yet negligibly shorten its length.

Although the above estimates are somewhat rough, they do reinstate squirt-flow as a plausible mechanism of seismic dissipation, so long as it operates within fracture interspace. There remains the question of what constitutes a "smallest pore", since laboratory investigations have traced the effectiveness of squirt-flow to only a preferential fraction of the pore-size distribution [e.g. $10^{-3} - 10^{-4}$ (Jones, 1986); $10^{-4} - 10^{-5}$ (Murphy et al., 1986)]. An answer can be obtained by considering the molecular diameter of H_2O ($\sim 3.2 \text{ \AA}$) and estimating the minimal crack thickness which accommodates enough monolayers so as to form a macroscopic liquid lattice. The lower bound $d_c \sim 10^{-8} \text{ m}$ is equivalent to $100 \text{ \AA}$; allowing two monolayers of bounded liquid leaves about 30 monolayers, enough to form a thin-film of free liquid. A remarkable experiment by Israelachvili (1986), in which thin-films of water were sinusoidally squeezed between two mica surfaces, has shown that liquid-water retains its bulk viscosity value down to at least $50 \text{ \AA}$, requiring only a single monolayer of bounded fluid. If such thin-films undergo Poiseuille flow in responding to seismic stressing at their confining walls, severe energy absorption can be expected. Israelachvili's result raises the possibility that cracks of aspect-ratio approaching 10^{-6} may be the preferential site of seismic dissipation by squirt-flow.

2.4 Conclusion

Several conclusions in favour of the occurrence of seismic dissipation by viscous fluid-flow in the crust can be drawn from the above analysis; at the same time, limitations have also become apparent. From a seismological

standpoint, it appears possible to discriminate frequency-independent from bell-shaped frequency-dependent depth-distributions of attenuation, provided that the total attenuation at the surface is observed over at least three frequency-decades. Realistic attenuation models must include both intrinsic and scattering causative mechanisms in a stratified structure, which can be explored, albeit approximately, within the framework of surface-wave attenuation theory and from the viewpoint that $Q_t^{-1}(\omega)$ mainly reflects dissipation.

Estimates of the squirt-flow characteristic frequency, observed in the kHz range in rock-samples, yield seismic values nearing one Hz when constrained by fracture-wall roughness field-measurements and crustal permeability data deduced from reservoir-induced seismicity. Therefore, the squirt-flow mechanism cannot be rejected as a plausible cause for the observed peak in crustal $Q_t^{-1}(\omega)$ data. However, these estimates strictly hold for the brittle portion of the crust. Although squirt-flow may still operate in a deep, ductile lower crust with partial melt-like fluids, it seems difficult to reconcile with the conjectured presence of brine-saturated lamellae of the type modelled by Hyndman & Shearer (1989). At any rate, seismic attenuation is inferred to be very low in the deep crust. On the other hand, if Bailey's (1990) mid-crustal fluid reservoir model in a brittle-like transitional zone is adopted, then squirt-flow seems promising because attenuation is inferred to be high there, particularly in a tectonically stable crust.

To evolve from a plausible to a probable mechanism, the magnitude of dissipation by squirt-flow, as modelled in the laboratory, must be adapted to fracture-porosity characteristic of crustal formations. The most advanced squirt-flow model is that developed by Murphy et al. (1986) for granular rocks; however, its physical and mathematical foundation is weak in that numerous assumptions are made leading to many parameters which are difficult to constrain and which may not all be relevant. A sound mathematical physics framework is necessary for estimation of the relative importance of possible parameters contributing to the energy absorption process. To this end, it is perhaps best to proceed by understanding the physics of dissipation at the characteristic-crack scale first, and then scale to crustal-layer dimensions.

2.5 References

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Chapter-3: Theory of fluid-film motion in a compliant crack¹

Dissipation of wave-energy by localised viscous flow of interstitial fluids is thought to dominate the intrinsic attenuation and dispersion response of saturated rock samples [Jones, 1986; Murphy, Winkler & Kleinberg, 1986]. This mechanism, generically referred to as "squirt-flow", has been modelled with various degrees of details, but always with the basic feature of two oscillating solid surfaces of large width, separated by a small layer of fluid. For the mechanism to be efficient at transforming deformational energy two conditions are necessary: the solid-fluid structure must generate pressure gradients under local stress; and the fluid must be very thin in order to generate viscous shearing as it flows back and forth in the narrow pore-space. In resonating rock samples, both conditions occur at grain contacts; in a seismically sampled crust, both are likely within rough fractures as I have suggested earlier.

Although viscous hydrodynamics of confined fluids has been developed for over a century in the study of lubrication, its application to squirt-flow has not been articulated except for the simplest cases, chosen for mathematical convenience in a rather *ad hoc* fashion. Thus, there are lingering doubts about the validity of several assumptions used in squirt-flow models as well as difficulties in the interpretation of the physics of dissipation, especially regarding whether inertial effects matter and whether flow is compressible or incompressible. If it is hoped to explain seismic dissipation in the crust on the basis of squirt-flow then, clearly, these concerns must be addressed.

A case in point is the acoustic-relaxation model of Murphy & al. (1986) in which the derivation of a compressible-flow governing equation appears inconsistent, because it is based on the Navier-Stokes equation for *incompressible* flow, and incomplete, because pore-wall lateral motion is ignored. Also problematic with this model is the carrying of inertial terms throughout the analysis, only to be summarily discarded *after* solving for imposed boundary conditions. Earlier models (O'Connell & Budiansky, 1977; Mavko & Nur, 1979) rely on *ad hoc* simplifying criteria which, although based on good physical arguments, are of questionable applicability to heterogeneous matter such as fractured rock-formations. The above criticism also applies to a related model of pore-fluid motion recently proposed by Dvorkin, Mavko & Nur (1992) in which fluid compressibility effects in reservoir-scale fractures were explored.

Hence, there is a clear lack in dissipation studies for a comprehensive *theory* of fluid-film motion in the most

basic configuration of cracked rock: a thin pore. I develop here — from first principles — a mathematically and physically consistent theory for the motion of a thin-film of compressible fluid in a compliant, narrow pore. This theory applies to both gases and liquids and goes beyond simply borrowing results from lubrication theory because compressible-liquid dynamics in constricted space is a seldom visited subject, presumably from lack of necessity in most practical applications. However, *gas-film* lubrication has been actively studied [e.g. Gross, 1980] which offers guidelines for developing a theoretical formulation that extends to liquid-films. The purpose of the theory is to provide a firm theoretical footing for squirt-flow models as well as to unify them under a generalised set of governing equations obtained with a *minimum* of assumptions.

I first define the scope of the theory by clarifying the rather counter-intuitive physics which arises when a very thin fluid of large breadth is set into both lateral and vertical motion by oscillating confining walls. Then, I develop the theory for fluid-films, starting from the general form of Navier-Stokes and continuity equations, and using an effective equation of state which depends solely on pressure. The essence of the analysis is the establishment of a non-dimensional governing equation from which an objective assessment of the relative importance of each term can be made, *prior* to solving for specified initial and boundary conditions. The resulting governing equation is compared with limiting cases available from lubrication theory and with models proposed to date for dissipation by local squirting. Boundary conditions are not considered here; they will be treated in their own right in chapter-4.

Except when referring to earlier work, the term "squirt-flow" will be avoided because it may convey the wrong idea that inertial effects dominate and may lead to turbulence. The tribological term "squeeze-flow" is more appropriate because it carries no such prejudice; although it implies forcing perpendicular to pore-walls, I shall not disregard in-plane forcing.

3.1 Squeeze-flow Physics

Figure-3.1 embodies the basic elements of oscillatory squeeze-flow: two nominally flat solid surfaces in relative motion, in contact with an adherent and viscous fluid-film. The most important aspect of the system is the smallness of the film's thickness relative to surface size. To be sure, an aspect-ratio of 10^{-5} is equivalent to a thickness of one mm over a distance of 100 meters. Common practice in viscous hydrodynamics consists in simplifying the general Navier-Stokes equations using generalised dimensionless groupings but, for such low aspect-ratio, it is not immediately obvious which fluid attributes can be neglected at the outset because several of these groupings lose their meaning. This is seldom realised.

Moreover, the situation initially appears more complex for crustal cracks of the type discussed in chapter-2 where the prevalent dimension is of the order of one mm length-wise which, for an aspect-ratio of 10^{-5} , corresponds to a thickness of 10^{-8} m. Since this is only 100 Å; therefore, close to the molecular scale, it appears unclear whether the continuum description implicit in hydrodynamics is

¹. An earlier version of this paper was presented at the fall meeting of the American Geophysical Union in San Francisco, California, on December 7th, 1992 (paper # S11B-04, Rouleau, P. M. A theory of seismic dissipation by squeeze-flow of fluid-films)

fitting.

Basic structural elements of squeeze-flow

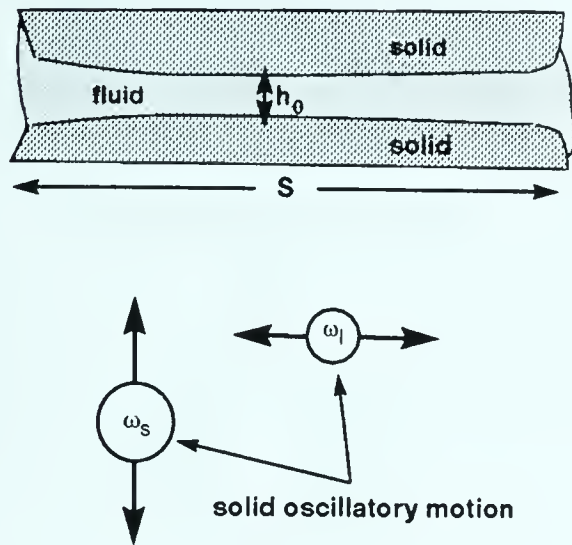


Figure-3.1 Basic elements of oscillatory squeeze-flow. Nominal thickness of viscous fluid film h_0 is much smaller than driving solid surfaces breadth S . Solid parts are in relative motion, laterally at frequency ω_l , vertically at frequency ω_s . Sketch is NOT drawn to scale, $h_0 \ll S$.

However, in a recent experiment on the viscosity of thin-film liquids, Israelachvili (1986) demonstrated that a sinusoidally squeezed film of water retains its macroscopic viscosity value down to thicknesses of at least 50 Å, and that bounded-water thickness is of the order of roughly one monolayer (~ 3 Å). Therefore, a continuum approach seems justified for water-like liquids down to film thicknesses of a few tens of Å, at least in a first approximation. Initial questions to be addressed are whether the flow is predominantly compressible or incompressible, and whether an adiabatic or isothermal regime prevails. The conventional criterion for distinguishing compressible from incompressible flow is Mach's number, which gives the ratio of fluid local velocity to sound velocity but, in the present situation, it is quite irrelevant because it is based on Bernoulli's principle which assumes *inviscid* flow. Development of a Bernoulli zone in a narrowly confined space is not a reasonable expectation.

Other arguments relying on waves generated from an impulse at one end of a long compliant tube are of dubious applicability because squeeze-flow motion is mainly generated by rigid oscillation of the confining surfaces. Considerations based on wave-propagation also assume, implicitly, adiabatic conditions; whilst these may prevail for solid parts, it is uncertainly so for a thin layer of interstitial fluid because high thermal conductivities of solid surfaces likely prevents significant temperature variations in the film due to adiabatic compression. Indeed, for most rocks, thermal conductivities are several times larger than for water and other liquids of interest [cf. Angenheister, 1982]]. Thus, given the large surface area of the film, isothermal conditions may prevail in the film².

As to the nature of the flow regime — laminar or turbu-

lent — the well-known Reynolds number, the ratio of the product of density, fluid velocity and channel width over viscosity, is an unequivocal discriminating factor. However, even if flow is laminar, it does not necessarily follow that compressibility is negligible as commonly assumed for a fluid moving "very slowly". Taylor & Saffman (1957) have shown that for low Reynolds numbers — and despite a Mach number much smaller than one — fluid compressibility had indeed a dominant effect on the pressure field observed between two moving disks separated by a thin layer of air.

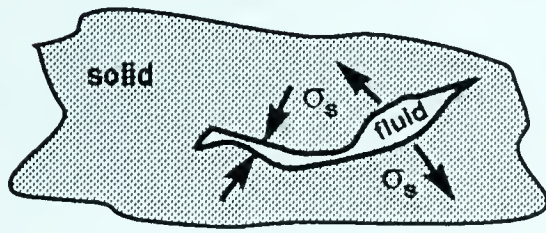
Another relevant criterion for constraining the flow nature is the size of the viscous skin-depth which evolves as a submerged plate sinusoidally moves parallel to its plane. It is used to estimate the extent of vorticity associated with shearing flow near the plane and the degree with which this vorticity is affected by convection. Defined as the square root of twice the ratio of kinematic viscosity to frequency, the skin-depth illustrates whether inertial flow is significant for a given geometry. Although this criterion, when applied to crustal cracks, suggests that inertia is negligible at seismic frequencies, it remains partially relevant as it is deduced strictly from pure lateral motion of a flat fluid-solid surface. Thus, there are limits to the discriminating efficiency of generalised non-dimensional groups used in a classical hydrodynamic approach.

3.1.1 Micro-mechanical approach

A more enlightening viewpoint is to regard the fluid-solid structure as a micro-mechanical system, in which solid-solid contact points are lumped into a single stiffness symbolically represented by a spring, and fluid compressibility and viscosity are represented, respectively, by a spring and viscous dashpot, as shown in fig-3.2. This arrangement is a representation of the standard linear solid used in linear viscoelasticity. Whilst a description in these terms is sometimes criticised for being vague on the manner in which internal parameters of a system produce the lag of strain behind stress, it is less ambiguous for squeeze-flow since fluid viscosity and matter compressibilities are not obscure quantities. The viscoelasticity approach is particularly suitable here when considering that departure from perfect elasticity are "slight" for most earth materials; therefore, justifiably amenable to a description in terms of both a linear and reversible process. This, combined with the complex theory formulation of viscoelasticity, provides a basis upon which an equivalence between the imaginary part of the system's response and dissipation by viscous flow can be defined. Of course, this viewpoint constrains the physics to be linear in the sense that stress must be proportional to strain-rate, but evidence in favour of such behaviour is convincing inasmuch as normal seismic strain levels ($\sim 10^{-8}$) are concerned [cf. chapter-1]. Additional advantages of linear viscoelasticity are that both dissipation and dispersion are obtained in the final analysis, and both isothermal and adiabatic descriptions are isomorphic, leaving interpretation of moduli a matter of choice [e.g. Christensen, 1971]

² For relevant bulk moduli differences between adiabatic and isothermal values are slight.

Fluid motion in constricted space



micro-mechanical analogue

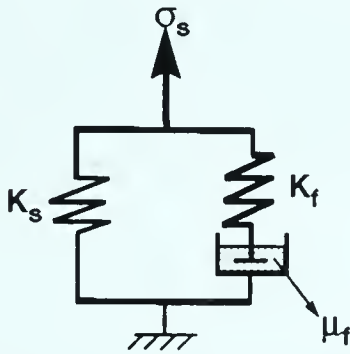


Figure-3.2 A representation of the standard linear solid as a micro-mechanical analogue of viscous dissipation to explore the phenomenology of squeeze-flow. σ_s is localised stress, K_f fluid bulk modulus, μ_f fluid viscosity, and K_s effective modulus of discrete solid contacts between confining surfaces.

It is well to emphasise that resistance of the fluid to wall motion takes the form of excess pressure, and that this pressure needs not be "large" to produce an appreciable effect. For typical values of seismic strain ($\sim 10^{-8}$) and crustal rock stiffness (~ 40 GPa), excess stress associated with a passing wave is roughly 400 Pa and is six orders of magnitude smaller than ambient stress at mid-crustal levels. Since moderately viscous fluids cannot sustain shear stresses for very long, flow will occur however small local differential stresses are and, consequently, viscous losses will follow.

With this in mind and using fig-3.2 for a *gedanke* experiment much of the physics to address in a theory of fluid motion in a low aspect-ratio pore comes into grasp. Upon application of compressive stress, the system immediately reacts by contraction of both solid-material and fluid-lattice: this is the instantaneous part of the response. Because of the link with the viscous-fluid dashpot, there is also a time-dependent part. In an harmonic stress situation, the question arises as to the behaviour of the film in between the two extremes of slow and rapid forcing. At fast rate, the motion of the fluid is fully resisted by its own viscosity; the system's response is then unrelaxed and characterised by adding fluid and solid stiffnesses. At slow rate, the fluid-film has ample time to flow around the piston without appreciably compressing; the system is then in a relaxed state and is characterised by the solid stiffness. Clearly, in between these states, the film can be expected to both compress and flow, with the combined effect producing a maximum delay in the overall strain-response, at a characteristic frequency. There lies the interest in explicitly computing the pressure field in the complex domain so that the imag-

inary part of the response, to which dissipation is attached, can be interpreted in terms of fluid attributes.

Important aspects of film squeezing can be visualised with the help of fig-3.3, which shows the connection between vertical motion of an approaching surface and horizontal motion of interstitial matter. In practice, however, flow outward of the gap will be resisted at the channel exit, particularly if the ambient fluid is confined to a small space, which motivates once more taking compressibility into account. Of course, in a real situation the flow is not that simple but it is amenable to a good degree of characterisation by way of hydrodynamics and, consequently, so is characterisation of dissipation by viscous squeeze-flow. The micro-mechanical point of view illustrates important physics for this purpose and improves on the use of *ad hoc* criteria.

3.2 Theory of thin-film motion

A theory of thin-film motion in a compliant pore must establish governing equations for the internal fluid-pressure field generated by oscillating solid-surfaces as these respond to locally applied stresses. To associate an imaginary impedance with the pore-fluid structure, frequency must appear in the formulation. Taking the viscoelasticity approach, constitutive equations relating stress to strain-rates are linear, but this follows naturally from the Newtonian behaviour of relevant fluids for conditions of interest. Therefore, the appropriate momentum relations are the Navier-Stokes equations.

All contributing factors must be retained, though the Navier-Stokes equations will be simplified by integrating over thickness. Compressibility and viscosity are the most important factors. The nature of flow onset makes Stokes' hypothesis dubious, so *both* coefficients of viscosity (shear and bulk) must be retained. Compressibility can be introduced via the continuity equation along with an equation of state expressing the relationship between density and pressure. In gas-film lubrication, the perfect gas law is used with density directly proportional to pressure, as in isothermal conditions (cf. Gross, 1980).

Squeeze-film analogue

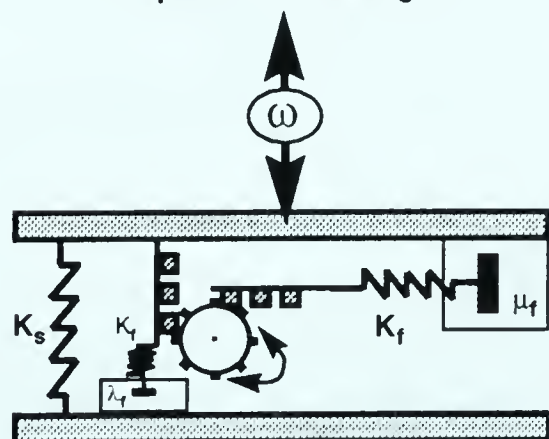


Figure-3.3 A mechanical analogue of squeeze-film illustrating the connection between vertical motion of an approaching solid surface and horizontal motion of the piston to which it is coupled (at the right). Sinusoidal motion of the upper surface causes both pistons to oscillate accordingly due to the gearing arrangement shown.

but with a phase lag imparted by the viscous dashpots. λ_f and μ_f are bulk and shear viscosity of the fluid, respectively. K_s is the effective stiffness of solid contacts, K_f the fluid's stiffness. ω is frequency of forcing

For liquids, I shall use the equation of state formulated by Muskat (1946):

$$\rho = \rho_0 e^{\beta p} \quad [3.1]$$

where ρ is density, ρ_0 reference density, and β (< 1) isothermal liquid compressibility. Since density depends only on pressure the need for an energy equation is circumvented, which would not be possible for adiabatic conditions. This situation is not too restrictive: Gross (1980) mentions a more general equation of state for liquids given by

$$\frac{d\rho}{\rho} = \frac{dP}{K_l} - \alpha' d\theta \quad [3.2]$$

where K_l is liquid bulk modulus, α' coefficient of thermal expansion, and θ temperature. For representative perturbation stress and water bulk modulus, the pressure term is of order 10^{-7} ; given that the coefficient of thermal expansion of water is itself of order 10^{-7} per Celsius degree, the temperature variation required for a significant density reduction relative to pure adiabatic compression is unacceptably high. Therefore, for conditions of interest, neglecting the temperature term is justifiable as is interpreting $K_l = 1/\beta$ in Eq-3.1 akin to an adiabatic bulk modulus.

The theory must lead to an informative governing equation. This is best accomplished by ensuring that all variables are of order unity so that the relative importance of each term of the equation becomes transparent from their coefficients. The aim is, then, to express the thin-film flow governing equation in dimensionless form.

3.2.1 Mathematical foundation of thin-film motion

Langlois (1962) carried-out the most thorough mathematical analysis of gas-film lubrication, in which the fewest assumptions are made about the predominant nature of the flow regime (common treatments of viscous fluid flow eliminate inertia *ab initio*). Langlois used the perfect gas law to change density into pressure at the very beginning of his analysis in order to develop a governing equation entirely in terms of pressure. This limits the applicability of his results to thin-films of isothermal gases. Here, I develop a more general theory in that density is allowed to be a more complicated function of pressure, as befits a fluid in the liquid state for example. In my analysis, the function relating density to pressure remains unspecified and so is applicable to any equation of state in which pressure is the independent variable. Other than these extensions, the mathematical development described here is similar to Langlois, albeit given in more detail.

The fluid in the film obeys the continuity relation and

its motion is governed by the Navier-Stokes equation, including both viscosity coefficients. The *proper* Navier-Stokes equation — with a minimum of assumptions implied — is given by Serrin (1959, p. 237) as:

$$\rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{F} - \nabla P + \quad [3.3]$$

$$\nabla [\lambda (\nabla \cdot \mathbf{v})] + \nabla \cdot (2\mu \mathbf{D})$$

where t is time, $\mathbf{v}$ velocity, $\mathbf{F}$ external (body) forces, $\mathbf{D}$ deformation tensor, and μ and λ are coefficients of shear and bulk viscosity, respectively. D/Dt is the material derivative. For ease of integration over a preferential coordinate, it is best to separate the governing equation into its components. Using the definition of the deformation tensor

$$D_{i,j} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad [3.4]$$

with $\{i, j\} = \{1, 2, 3\}$, and expressing operators in component form, Eq-3.3, reads

$$\rho \frac{Dv_i}{Dt} = \rho F_i - \frac{\partial P}{\partial x_i} + \lambda \frac{\partial^2 v_j}{\partial x_i \partial x_j} + \mu \left(\frac{\partial^2 v_i}{\partial x_j \partial x_j} + \frac{\partial^2 v_j}{\partial x_i \partial x_j} \right) + \quad [3.5]$$

$$\frac{\partial \lambda}{\partial x_i} \frac{\partial v_j}{\partial x_j} + \frac{\partial \mu}{\partial x_j} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

with D/Dt defined by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \quad [3.6]$$

The continuity equation is given by Serrin [1959, p132] as

$$\frac{D\rho}{Dt} + \rho \frac{\partial v_j}{\partial x_j} = 0 \quad [3.7]$$

which can be written

$$\frac{\partial v_j}{\partial x_j} = -\Delta \quad [3.8]$$

with the dilation Δ defined as

$$\Delta = \frac{1}{\rho} \frac{D\rho}{Dt} \quad [3.9]$$

In principle, viscosity coefficients λ and μ are functions of the state variables but they can realistically be taken as constant for relevant fluids and pressures. A further simplification follows from eliminating external (body) forces whose effects here are infinitesimal relative to pressure gradients. Thus, the first and last two terms on the right of Eq-3.5 vanish, yielding the *isoviscous* relation

$$\rho \frac{Dv_i}{Dt} = - \frac{\partial P}{\partial x_i} + \lambda \frac{\partial^2 v_j}{\partial x_i \partial x_j} + \mu \left(\frac{\partial^2 v_i}{\partial x_j \partial x_j} + \frac{\partial^2 v_j}{\partial x_i \partial x_j} \right) \quad [3.10]$$

Using the dilation Δ given by Eq-3.8, the governing equation

tion becomes

$$\rho \frac{Dv_i}{Dt} = -\frac{\partial}{\partial x_i} [P + (\lambda + \mu) \Delta] + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j} \quad [3.11]$$

This momentum equation, relating velocity, pressure and density, embodies the foundation of compressible fluid-film hydrodynamics. The aim of the mathematical development is to eventually eliminate velocity and so obtain an equation in P and ρ , in which density can be expressed in terms of pressure via the equation of state. This can be accomplished by integrating the continuity equation over the film's thickness, using a reduced form of Eq-3.11 for evaluating the ensuing velocity integrand. Reduction of Eq-3.11 follows from a perturbation analysis in terms of dimensionless quantities and from using adherence conditions at solid-fluid boundaries (i.e the "no-slip" condition).

3.2.2 Definition of dimensionless quantities for fluid-film dynamics

Figure-3.4 shows the cartesian coordinate system used as reference for developing the governing equation, along with two surfaces everywhere separated by a thin-film. Points on surfaces are located at

$$x_3 = S_a(x_1, x_2, t) \quad [3.12]$$

and

$$x_3 = S_b(x_1, x_2, t) \quad [3.13]$$

leading to the definition of the film's thickness as

$$h(x_1, x_2, t) = S_b(x_1, x_2, t) - S_a(x_1, x_2, t) \quad [3.14]$$

When the lower and upper surfaces move their velocities are, respectively, ${}_aV_i$ and ${}_bV_i$. These three-component velocities, taken relative to the ambient fluid, obey the kinematic constraints:

$${}_aV_3 = \frac{\partial S_a}{\partial t} + {}_aV_1 \frac{\partial S_a}{\partial x_1} + {}_aV_2 \frac{\partial S_a}{\partial x_2} \quad [3.15]$$

$${}_bV_3 = \frac{\partial S_b}{\partial t} + {}_bV_1 \frac{\partial S_b}{\partial x_1} + {}_bV_2 \frac{\partial S_b}{\partial x_2} \quad [3.16]$$

In order to exploit the low aspect-ratio characteristic of thin-films it is useful to separate horizontal and vertical spatial coordinates, and to normalise them using representative surface size S for both horizontal directions and nominal film thickness h_0 for the vertical direction. The ratio

$$\epsilon = \frac{h_0}{S} \quad [3.17]$$

constitutes a perturbation parameter. The indicial convention used here is: $i, j = \{1, 2, 3\}$; and $k, l = \{1, 2\}$.

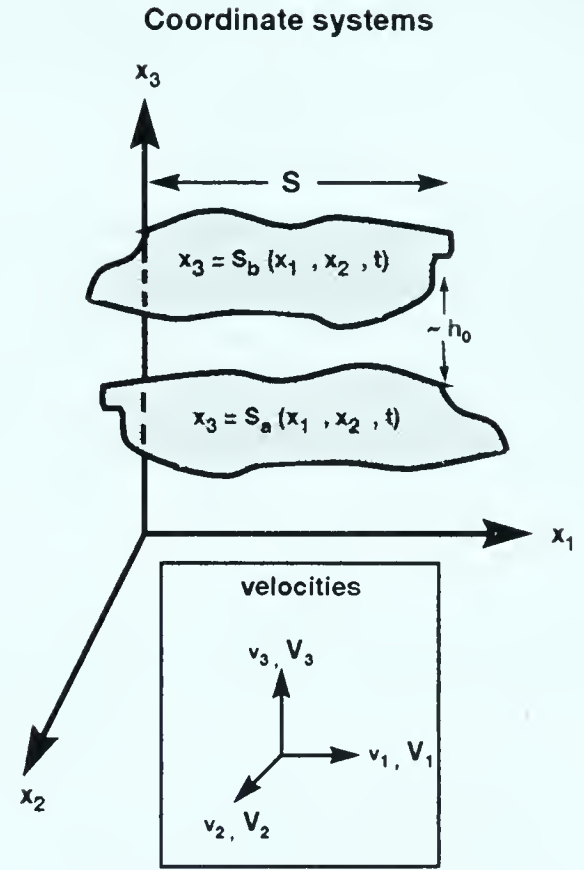


Figure-3.4 Coordinate systems used for developing the fluid-film governing equation. The film, of nominal thickness h_0 , flows in between two solid surfaces of characteristic breadth S , with $h_0 \ll S$.

Thus, normalised coordinates are defined by

$$z = \frac{x_3}{h_0} = \frac{x_3}{\epsilon S} \quad [3.18]$$

and

$$X_k = \frac{x_k}{S} \quad [3.19]$$

Insertion of these in Eq-3.11 and re-arrangement yields: for horizontal components,

$$\mu \frac{\partial^2 v_k}{\partial z^2} = \epsilon^2 \left\{ -\mu \frac{\partial^2 v_k}{\partial X_l \partial X_l} + S \frac{\partial}{\partial X_k} [P + (\lambda + \mu) \Delta] + S^2 \rho \frac{Dv_k}{Dt} \right\} \quad [3.20]$$

and for the vertical component,

$$\mu \frac{\partial^2 v_3}{\partial z^2} = \epsilon S \frac{\partial}{\partial z} [P + (\lambda + \mu) \Delta] - \epsilon^2 \left[\mu \frac{\partial^2 v_3}{\partial X_k \partial X_k} - S^2 \rho \frac{Dv_3}{Dt} \right] \quad [3.21]$$

Variables P , t , v_i , and ρ , as well as surface velocities ${}_aV_i$ and ${}_bV_i$ can be expressed in dimensionless form by scaling, accordingly to the normalisation and notation shown

in Table-3.1. Time t is naturally referenced to ω , a typical frequency associated with the period τ of vertical oscillation of surfaces S_a, S_b such that:

$$T = \omega t \quad [3.22]$$

where T is dimensionless time. Two reference velocities, V and ωh_0 , can be associated with lateral and vertical components of surface motion, respectively; thus, with surface label $m = \{a, b\}$, the dimensionless surface velocities of unit order are:

$${}_m U_k = \frac{{}_m V_k}{V} \quad [3.23]$$

and

$${}_m W = \frac{{}_m V_3}{\omega h_0} \quad [3.24]$$

squeezing, with P varying as $\mu \omega \epsilon^{-2}$. This scaling is equally applicable to compressible liquids, yielding the normalised pressure

$$\bar{P} = \frac{P}{\mu (\omega + V/S) \epsilon^{-2}} \quad [3.26]$$

This normalisation reflects the phenomenon, well-documented in lubrication engineering, that pressure in a narrowly constricted fluid arises from shear viscosity of the film, with a value proportional to the product of viscosity by a measure of the rate of deformation (e. g. Pinkus & Sternlicht, 1961).

Finally, the density can simply be scaled by its ambient value ρ_0 , giving

$$\bar{\rho} = \frac{\rho}{\rho_0} \quad [3.27]$$

The normalisation of Table-3.1 can now be used to establish the momentum and continuity equations for

Table 3.1: Normalisation and notation

Quantity	Dimensional	Normalisation	Non-dimensional
Time	t	$t = 1 / \omega$	T
horizontal surface velocity	${}_m V_k$	V	${}_m U_k$
vertical surface velocity	${}_m V_3$	$h_0 \omega$	${}_m W$
horizontal fluid velocity	v_k	$\omega + V/S$	u_k
vertical fluid velocity	v_3	$h_0 \omega$	w
pressure	P	$\mu (\omega + V/S) \epsilon^{-2}$	P
density	ρ	ρ_0	$\bar{\rho}$

Nota: $k = \{1, 2\}$; $m = \{a, b\}$

Considering that the fluid adheres to its bounding surfaces, the temporal scales — implicit in the definition of V and ωh_0 — may be viewed in terms of two reciprocal times: V/S , a fluid shearing-rate caused by lateral motion of solid surfaces; and ω , a characteristic frequency of squeezing motion. Vertical reference velocity ωh_0 can also be used to normalise the vertical component of fluid velocity v_3 , giving w as the dimensionless velocity of unit order.

Because both shearing and squeezing operate in the film, fluid velocities in the x_1 - x_2 directions are best normalised accordingly to

$$u_k = \frac{v_k}{\omega + V/S} \quad [3.25]$$

with the expectation that u_k is of order unity.

In his development of the gas-film governing equation, Langlois (1962) normalised pressure by combining two limiting cases of incompressible fluid motion: one in pure shearing, with P varying as $(\mu V / S) \epsilon^{-2}$; the other in pure

fluid-films in dimensionless form and so allow a proper perturbation analysis on the basis of parameter ϵ , without any prejudice about the predominance of inertial or viscous regimes.

3.2.3 Perturbation analysis

Equations [3.20] and [3.21], giving horizontal and vertical components of fluid motion, are to be expressed in dimensionless form, with terms weighted by perturbation parameter ϵ to an appropriate power. Here, the strategy is to eliminate second and higher-order terms in ϵ in the three momentum equations, and then accordingly adjust the continuity equation which is coupled to them. This approach is quite valid in view of the smallness of the ϵ 's under consideration (at most 10^{-2}). In order to clarify the steps leading to the perturbation equation, scaled material derivative D/Dt , dilatational stress $(\lambda + \mu) \Delta$, and kinematic constraints Eq-3.15 and Eq-3.16 are developed in the Appendix. Thus, insertion of the scaling quantities in Eq-3.20 and Eq-3.21 yields: for the horizontal components of fluid motion,

$$\frac{\partial \bar{P}}{\partial X_k} = \frac{\partial^2 u_k}{\partial z^2} - R_s \bar{\rho} \left[\frac{\partial u_k}{\partial T} + w \frac{\partial u_k}{\partial z} \right] - (R_s + R_l) \bar{\rho} \left[u_k \frac{\partial u_k}{\partial X_k} \right] - \epsilon^2 \left\{ \frac{1}{(1 + \frac{V}{\omega S})} \frac{\partial \Delta_s}{\partial X_k} + \frac{1}{(1 + \frac{\omega S}{V})} \frac{\partial \Delta_l}{\partial X_k} - \frac{\partial^2 u_k}{\partial X_l \partial X_l} \right\} \quad [3.28]$$

and for the vertical component

$$\begin{aligned} \frac{\partial \bar{P}}{\partial z} &= \frac{\epsilon^2}{(1 + \frac{V}{\omega S})} \times \\ &\left(\frac{\partial^2 w}{\partial z^2} - \frac{\partial \Delta_s}{\partial z} - R_s \bar{\rho} \left(\frac{\partial w}{\partial T} + w \frac{\partial w}{\partial z} \right) - \right. \\ &\quad \left. (R_s + R_l) \bar{\rho} u_k \frac{\partial w}{\partial X_k} \right) \\ &\quad - \frac{\epsilon^2}{(1 + \frac{\omega S}{V})} \frac{\partial \Delta_l}{\partial z} + \frac{\epsilon^4}{(1 + \frac{V}{\omega S})} \frac{\partial^2 w}{\partial X_k \partial X_k} \end{aligned} \quad [3.29]$$

where Δ_s , Δ_l are dimensionless quantities comprising convective and inertial density terms — weighed by bulk-to-shear viscosity ratios — and associated with the rate of deformation in squeezing and in lateral mode, respectively (see Appendix, Eq-A.4 and Eq-A.5). R_s and R_l are Reynolds numbers for squeezing (s) and lateral (l) motion arising from the factorisation of ϵ and defined as

$$R_s = \frac{\rho_0 h_o^2 \omega}{\mu} \quad [3.30]$$

$$R_l = \frac{\rho_0 h_o^2 V}{\mu S} \quad [3.31]$$

Examination of Eq-3.28 and Eq-3.29 indicates that all terms containing bulk viscosity λ are of second order. More importantly, Eq-3.29 shows that the vertical pressure gradient only depends on second order perturbation; therefore, to first order, pressure across the film's thickness can be taken as constant, giving

$$\frac{\partial \bar{P}}{\partial z} = 0 \quad [3.32]$$

Consistently with first order approximation, the horizontal equation of motion becomes

$$\begin{aligned} \frac{\partial \bar{P}}{\partial X_k} &= \frac{\partial^2 u_k}{\partial z^2} - R_s \bar{\rho} \left[\frac{\partial u_k}{\partial T} + w \frac{\partial u_k}{\partial z} \right] - \\ &\quad (R_s + R_l) \bar{\rho} \left[u_k \frac{\partial u_k}{\partial X_k} \right] \end{aligned} \quad [3.33]$$

in which only shear viscosity μ is implicit and in which all convective and inertial terms are weighted by the Reynolds numbers.

Turning now to the continuity relation, Eq-3.7, its

dimensionless form is found by scaling as above, but with the simplifying constraint that the vertical density gradient now vanishes in accordance with the zero vertical pressure gradient (Eq-3.32). As shown in the Appendix, the continuity equation in dimensionless form reduces to:

$$\frac{1}{(1 + \frac{V}{\omega S})} \frac{\partial}{\partial T} (\bar{\rho} H) + \frac{\partial}{\partial X_k} \left(\bar{\rho} \int_{\bar{a}}^{\bar{b}} u_k dz \right) = 0 \quad [3.34]$$

where $H = h / h_0$ and where $\bar{a} = S_a / h_0$, $\bar{b} = S_b / h_0$, thereby indicating integration over the film's thickness.

Dimensionless equations [3.33] and [3.34] can now be combined into a single governing equation for thin-film flow, provided a suitable expression for the integrand u_k of Eq-3.34 can be found.

3.2.4 Dimensionless governing equation for thin-films

Eq-3.33, expressed in terms of the second derivative of fluid velocity, reads

$$\begin{aligned} \frac{\partial^2 u_k}{\partial z^2} &= \frac{\partial \bar{P}}{\partial X_k} + R_s \bar{\rho} \left[\frac{\partial u_k}{\partial T} + w \frac{\partial u_k}{\partial z} \right] + (R_s + R_l) \bar{\rho} \times \\ &\quad \left[u_k \frac{\partial u_k}{\partial X_k} \right] \end{aligned} \quad [3.35]$$

where it is apparent that $\partial^2 u_k / \partial z^2$ is a function of velocity u_k and coordinate z , i-e

$$\frac{\partial^2 u_k}{\partial z^2} = F(u_k, z) \quad [3.36]$$

In order to find a representation for integrand u_k of Eq-3.34, I used an integral equation technique in which Eq-3.35 is twice integrated so as to yield a Fredholm integral of the form

$$u_k(z) = g_k(z) + \int_A^B K(z, s) F[s, u_k(s)] ds \quad [3.37]$$

where $g_k(z)$ is a function containing the two constants of integration and $K(z, s)$ is a kernel function splitting the integral accordingly to the position of dummy variable s in interval $[A, B]$. The two constants of integration are specified by the condition of adherence at fluid-solid surfaces, given in dimensionless form by

$$u_k|_{z=\bar{a}} = \frac{a U_k}{1 + \frac{\omega S}{V}} \quad [3.38]$$

$$u_k|_{z=\bar{b}} = \frac{b U_k}{1 + \frac{\omega S}{V}} \quad [3.39]$$

Applying the Fredholm integral method to Eq-3.35 for

obtaining an explicit form of $u_k(z)$ is a lengthy but straightforward operation; the result is the integro-differential equation

$$u_k(z) = \frac{1}{2} \frac{\partial \bar{P}}{\partial X_k} (z - \bar{a})(z - \bar{b}) + \frac{{}_a U_k \bar{b} - {}_b U_k \bar{a} + ({}_b U_k - {}_a U_k)z}{(1 + \frac{\omega S}{V})(\bar{b} - \bar{a})} + \frac{1}{2} \bar{\rho} \int_{\bar{a}}^{\bar{b}} K(z, s) \left((R_s + R_l) u_l \frac{\partial u_k}{\partial X_l} + R_s \left(\frac{\partial u_k}{\partial T} + w \frac{\partial u_k}{\partial s} \right) \right) ds \quad [3.40]$$

Inserting this representation of $u_k(z)$ in Eq-3.34 and integrating, I obtain

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial (\bar{\rho} H)}{\partial T} + \frac{6}{(1 + \frac{\omega S}{V})} \frac{\partial}{\partial X_k} [\bar{\rho} H ({}_a U_k + {}_b U_k)] - \frac{\partial}{\partial X_k} \left(\bar{\rho} H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s) \quad [3.41]$$

where I_l, I_s are non-linear inertia factors defined by

$$I_l = 6 \frac{\partial}{\partial X_k} \left\{ \bar{\rho}^2 \int_{\bar{b}}^{\bar{a}} \left[(z - \bar{a})(z - \bar{b}) \left(u_l \frac{\partial u_k}{\partial X_l} \right) \right] dz \right\} \quad [3.42]$$

$$I_s = I_l + 6 \frac{\partial}{\partial X_k} \left\{ \bar{\rho}^2 \int_{\bar{b}}^{\bar{a}} \left[(z - \bar{a})(z - \bar{b}) \left(\frac{\partial u_k}{\partial T} + w \frac{\partial u_k}{\partial z} \right) \right] dz \right\} \quad [3.43]$$

Eq-3.41 is a dimensionless, non-linear integro-differential equation for the motion of a compressible fluid of nominal aspect-ratio ϵ . It is correct to first order in ϵ and based on *four* assumptions: the fluid is non-gravitating, isoviscous, adherent to solids, and its density depends only on pressure. Each term is of unit order; therefore, coefficient magnitudes attest of their relative importance.

3.3 Squeeze-flow of fluid-films

Eq-3.41 holds for both gases and liquids inasmuch as their densities obey a law of the form $\rho(P)$, ρ being of order unity. That this is so for gases can be seen by using the perfect gas law expressed as $\rho = P/P_{\text{ambient}}$ in Eq-3.41, giving the pressure equation

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial}{\partial T} (\bar{P} H) + \frac{6}{(1 + \frac{\omega S}{V})} \frac{\partial}{\partial X_k} [\bar{P} H ({}_a U_k + {}_b U_k)] - \frac{\partial}{\partial X_k} \left(\bar{P} H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s) \quad [3.44]$$

Except for notation, this equation is identical to that found by Langlois (1962) who set density equal to pressure at the start.

For liquids, $\bar{\rho} = e^{\beta P}$ by Eq-3.1, yielding the more complicated pressure equation

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial}{\partial T} (H e^{c\beta(\bar{P})}) + \frac{6}{(1 + \frac{\omega S}{V})} \frac{\partial}{\partial X_k} [H e^{c\beta(\bar{P})} ({}_a U_k + {}_b U_k)] - \frac{\partial}{\partial X_k} \left(H^3 e^{c\beta(\bar{P})} \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s) \quad [3.45]$$

where c is a constant given by Eq-3.26, and where I_l and I_s now contain the exponential pressure function.

In both Eq-3.44 and Eq-3.45 inertial and non-linear (convective) terms appear on the right, with Reynolds numbers as coefficients. When these are small, Eq-3.44 and 3.45 can be set to zero; in particular, Eq-3.41 becomes the generalised Reynolds³ equation in dimensionless form. For crustal liquids, R_s and R_l estimates are quite small at seismic frequencies, even for shallow cracks [e.g. $R_s \sim 10^{-2}$ at most]. For gases, R_s and R_l may be of order unity but such fluids may not generate significant losses by shearing flow in view of their low viscosities. A mixture of gas and liquid would yield intermediate values of R_s and R_l , depending on the relative concentrations of the two phases.

It is tempting to conclude immediately that the right-hand side of Eq-3.41 can be nullified for crustal materials and seismic frequencies; however, it is prudent to examine conditions under which this could be unwise. Deletion from a dimensionless equation of terms with small coefficients is valid only if the weighted quantities remain of order unity. It is conceivable that the density gradients implicit in inertia factors I_s and I_l may exceed unit order for thin-films whose bounding surfaces are geometrically irregular. For instance, an upper wall with a pronounced protuberance would force flow-lines to channel in a much narrower space than nominal thickness, with consequential enhancement of dissipation from acute molecular friction or even from generation of eddies. This type of problems has no easy answers, although some elucidation is possible from examining Eq-3.41 as it stands.

Firstly, for the right-hand side of Eq-3.41 to be of unit order, inertia factors must reach $\sim 10^3$ for Reynolds num-

³ Reynolds (1886) derived his classic equation assuming incompressible flow and so without density entering the expression. Numerous forms of "Reynolds equation" exist in the literature.

bers $\sim 10^{-3}$; judging from density terms in a chain rule expansion of Eq-3.42 and Eq-3.43, density gradients would have to be unacceptably large for significantly viscous crustal fluids which may be only "slightly" compressible. As to convective terms, their magnitude could be estimated from observed velocity profiles around corners, but experimental evidence indicates that departure from laminar flow occurs at Reynolds numbers in excess of 1000 [cf. Schlitching, 1979]. From this, setting the right-hand side of Eq-3.41 to zero appears justified for the crustal environment.

This reasoning is somewhat facile, however, when considering that perturbation parameter ϵ and Reynolds numbers R_s and R_l are calculated using nominal dimensions of the film. Recent experimental and theoretical work on fluid flow within rough confinements has established that the overall flow rate is mainly controlled by the thinnest dimensions in the largest spaces available to circulation (e.g. Pyrak-Nolte & al., 1988). This prompts the question of which aspect-ratio to use for Reynolds numbers to be meaningful. Clearly, if the aspect-ratio is not scaling but biased high from the short length beneath a protuberance then the above analysis needs revision, not only because Reynolds numbers may be important but also because ϵ^2 terms in Eq-3.28/3.29 may be so magnified by large non-linear factors (Δ_s , Δ_l) that vertical pressure gradients become no longer negligible. Needless to say that this would add complexity to an already strenuous problem.

Despite these difficulties, some estimates of the effect of geometrical irregularities on thin-film flow can be made using recent results from numerical simulations of flow through rough channels, even though these results strictly apply to incompressible flow between immobile walls. Figure-3.5 helps in illustrating my reasoning, the argument being that dissipation by viscous shearing is intimately linked to the rate at which a Newtonian fluid flows and so to the factors affecting it. First considering small departures from smooth confining walls, the pressure field in the vicinity of spatially periodic grooves has been computed by Larrea & al. (1992) for channel height-to-groove spacing ratio of ~ 5 and for a range of Reynolds number pairs defined for the spacing and the channel. Strong pressure gradients were indeed found around corners, even for low Reynolds numbers, but when flow rates were compared to those predicted for Poiseuille flow (straight walls) they positively differed by at most six per cent. Thus, dissipation does not seem to be significantly augmented by moderate roughness and mild vorticity.

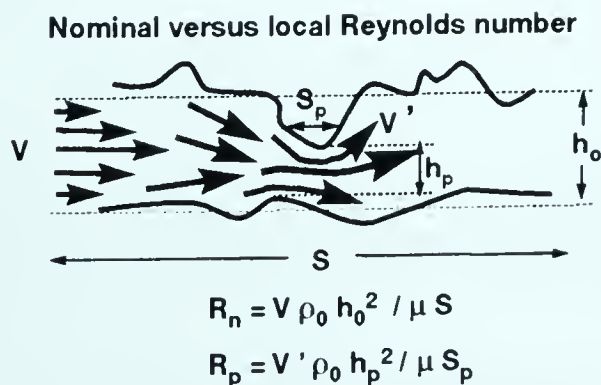


Figure-3.5 Visualisation of two-dimensional viscous flow within a rough-walled confinement. Nominal dimensions are h_0 and S for which unidirectional flow profile V is parabolic. R_n is the corresponding (nominal) Reynolds number. Protuberance dimensions are h_p and S_p for which flow profile V' is bidirectional. R_p is the corresponding (local) Reynolds number. Flow rate at the right end may be enhanced or diminished relative to parabolic flow.

More severe roughness has been considered by Zimmerman & al. (1991) who investigated, analytically, the effect of sinusoidal, sawtooth, and fractal walls on the permeability of a characteristic fracture — the controlling parameter of flow rate at fixed driving pressure. In particular, they have suggested a criterion for the validity of the Reynolds equation, given a measure of confining wall roughness. Using flow parallel to the wall as reference, they showed that permeability decreases with increasing roughness, taken proportional to the standard deviation σ_h from the mean gap topography h_m . Interestingly, departure from nominal permeability becomes significant (at the 10% level) only beyond $\sigma_h \sim 0.2 h_m$ and reaching a maximal reduction of one half when $\sigma_h \rightarrow h_m$, with little sensitivity to roughness details. Thus, at worst, dissipation in the gap could be reduced by a factor of two relative to flow parallel to the wall. Using scaling properties of fractal roughness and second order corrections to Reynolds equation, Zimmerman & al conclude that as long as the dominant spatial wavelength of roughness is larger than $5 \sigma_h$ the wall can be considered "smooth".

This latter condition is actually satisfied for natural fractures such as those reported by Scholz (1988) and others, and on which the concept of a characteristic crack explained in chapter-2 is based. Therefore, after a less superficial analysis, defining ϵ , R_s and R_l from full dimensions of the film (e. g. $\epsilon \sim 10^{-5}$), and not from protuberance dimensions, seems quite justified. It may be hoped then that, despite the likelihood of rough-walled cracks, the main features of dissipation by thin-film viscous-flow will be captured by assuming smooth confining surfaces.

3.3.1 Specialisation to squeeze-flow

In the context of this thesis, the above considerations justify neglect of inertia terms in Eq-3.41, in which case the governing equation of thin-film flow becomes

$$\begin{aligned}
 & \frac{12}{\left(1 + \frac{V}{\omega S}\right)} \frac{\partial}{\partial T} (\bar{\rho} H) + \\
 & \frac{6}{\left(1 + \frac{V}{\omega S}\right)} \frac{\partial}{\partial X_k} [\bar{\rho} H ({}_a U_k + {}_b U_k)] - \\
 & \frac{\partial}{\partial X_k} \left(\bar{\rho} H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = 0
 \end{aligned} \tag{3.46}$$

The implication of this equation for the motion of fluid-films encountered in practice can be examined by renor-

malising the pressure to ambient pressure p_0 , which allows a quantitative comparison of the two modes of dissipation: squeezing and in-plane forcing. Doing so yields:

$$\frac{\partial}{\partial X_k} \left(\bar{\rho} H^3 \frac{\partial \bar{p}}{\partial X_k} \right) = \sigma \frac{\partial}{\partial T} (\bar{\rho} H) + \Lambda \frac{\partial}{\partial X_k} [\bar{\rho} H ({}_a U_k + {}_b U_k)] \quad [3.47]$$

where $\bar{p} = P/p_0$, and σ, Λ are dimensionless groupings defined by

$$\sigma = \frac{12\mu S^2 \omega}{p_0 h_0^2} \quad [3.48]$$

and

$$\Lambda = \frac{6\mu SV}{p_0 h_0^2} \quad [3.49]$$

In lubrication engineering σ and Λ are called "squeezing" and "bearing" number, respectively, with $\Lambda \gg \sigma$ for a wide range of machine parts for which V is very large. In attenuation studies, however, this is not so; cracks filled with moderately viscous fluids and subjected to pure shearing generate little dissipation relative to pure squeezing, as shown by Mavko & Nur (1979) on the basis of energy considerations. This can be succinctly substantiated here by rewriting Eq-3.48 and Eq-3.49 as

$$\sigma = \frac{12\mu \omega}{p_0} \frac{1}{\varepsilon^2} \quad [3.50]$$

and

$$\Lambda = \frac{6\mu \omega_l}{p_0} \frac{1}{\varepsilon} \quad [3.51]$$

where $\omega_l = V / h_0$, which can be interpreted as a characteristic shearing-rate of lateral motion. For smooth confining walls $\omega_l \rightarrow \omega$, giving

$$\frac{\sigma}{\Lambda} \sim 2/\varepsilon \quad [3.52]$$

Since $\varepsilon \ll 1$, the squeezing mode clearly dominates the flow regime of thin-films; therefore, to a good approximation, the governing equation for thin-film motion reduces to:

$$\frac{\partial}{\partial X_k} (\bar{\rho} H^3 \frac{\partial \bar{p}}{\partial X_k}) = \sigma \frac{\partial}{\partial T} (\bar{\rho} H) \quad [3.53]$$

Hereon, Eq-3.53 will be referred to as the generalised "squeeze-flow" equation. With $\bar{p}$ an explicit function of pressure, Eq-3.53 becomes a non-linear partial differential equation of two dependent variables ($H, \bar{p}$), in three independent variables (X_1, X_2, T).

3.3.2 Recoverability of "squirt-flow"

models

The generality of the governing equation for thin-films, Eq-3.41, can be evaluated by examining the various forms it takes for problems treated in lubrication engineering and in attenuation studies of the "squirt-flow" mechanism. As mentioned earlier, if the film is an isothermal gas $\bar{\rho} = \bar{P}$ and Langlois' equation is recovered, as is Reynolds' generalised equation when the fluid is incompressible ($\bar{\rho} = 1$) and inertia and convective terms are set to zero.

Comparison with squirt-flow governing equations is not as straightforward because all are riddled with a variety of *ad hoc* assumptions. The most elementary form of squirt-flow is found in the work of O'Connell & Budiansky (1977) who defined a critical frequency for maximal dissipation in a saturated, cracked medium on the basis of the rate at which an incompressible fluid exits two parallel plates, one approaching the other at constant speed (cf. their appendix, equations A.20 and A.30). This hydrodynamic problem was solved by Stefan in 1874 (as reported by Bird et al., 1987) for the pressure between two coaxial disks of radius R_d distanced by $2h$, with the pressure reaching ambient at the periphery. The solution, obtained by successive approximations, is

$$P = \frac{3\mu R_d^2}{4h^3} \left(-\frac{dh}{dt} \right) \left[1 - \left(\frac{r}{R_d} \right)^2 \right] + p_0 \quad [3.54]$$

This model can be recovered from the squeeze-flow equation by first writing Eq-3.53 in cylindrical coordinates, which reads

$$\frac{\partial}{\partial R} (R \bar{\rho} H^3 \frac{\partial \bar{p}}{\partial R}) = R \sigma \frac{\partial}{\partial T} (\bar{\rho} H) \quad [3.55]$$

where $R = [X_1^2 + X_2^2]^{1/2} = r / S$. Cancelling $\bar{\rho}$ and expressing the result in dimensional form, this equation, for a film $2h$ thick, becomes

$$\frac{\partial}{\partial r} (r h^3 \frac{\partial p}{\partial r}) = 3\mu r \frac{\partial h}{\partial t} \quad [3.56]$$

Because h is only function of time, this expression reduces to

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{3\mu}{h^3} \frac{dh}{dt} \quad [3.57]$$

Insertion of Stefan's solution in this equation verifies the right-hand side identically, thereby showing the recoverability of the squirt-flow model implicit in O'Connell & Budiansky (1977).

Eq-3.55 can also be used to recover Murphy & al's (1986) governing equation for squirt-flow of a compressible liquid in and out of a disk-shape pore of low-aspect ratio. However, as mentioned in the introduction, Murphy & al. included inertia effects which I have shown to be unimportant in section 3.3.1 (from their data I find a small R_g). Fortunately, in their expression for the pressure-field governing equation, inertial effects are readily separable which allows a direct comparison with the pressure-field predicted here. Without inertial terms, their

equation (22), with all factors made explicit, reads

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - \frac{h_0}{K_l} \left(i\omega \frac{12\mu}{h_0^3} \right) p = i\omega \frac{12\mu}{h_0^3} \Delta h \quad [3.58]$$

where Δh is the amplitude of pore-wall oscillation ($\ll 1$) and K_l fluid bulk modulus.

For a meaningful comparison with Eq-3.58, Eq-3.55 must first be linearised. To perform the linearisation, I used two expansions: one for $\bar{p}$ from Eq-3.1, up to terms of order β^2 ; the other for $\bar{p}$, up to order ξ^2 , where ξ is the magnitude of solid surface displacement (dimensionless). Thus,

$$\bar{p} = 1 + \beta p_0 \bar{p} + O(\beta^2) \quad [3.59]$$

$$\bar{p} = \xi \bar{p}_1 + O(\xi^2) \quad [3.60]$$

Because walls are assumed parallel, H is a function of time only,

$$H = 1 + \xi e^{iT} \quad [3.61]$$

Inserting [3.59→3.61] in Eq-3.55, and neglecting second order terms, I obtain

$$\frac{\partial^2 \xi \bar{p}_1}{\partial R^2} + \frac{1}{R} \frac{\partial \xi \bar{p}_1}{\partial R} = \frac{\sigma}{H^3} (i\xi e^{iT} + \beta p_0 \frac{\partial \xi \bar{p}_1}{\partial T}) \quad [3.62]$$

which, for harmonic pressure $\bar{P}_1 = \xi \bar{p}_1 e^{iT}$, yields the steady-state equation

$$\frac{\partial^2 \bar{P}_1}{\partial R^2} + \frac{1}{R} \frac{\partial \bar{P}_1}{\partial R} - i \frac{\sigma}{H^3} \beta p_0 \bar{P}_1 = i \frac{\sigma}{H^3} \xi \quad [3.63]$$

In dimensional form, with $\bar{P}_1 = p / p_0$, this equation reads

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - \beta h_0 \left(i\omega \frac{12\mu}{h_0^3} \right) p = i\omega \frac{12\mu}{h_0^3} \varepsilon h_0 \quad [3.64]$$

which is identical to Eq-3.58 by recognising that $\beta = 1 / K_l$ and $\Delta h = \xi h_0$.

Murphy & al. derived their relation starting from Navier-Stokes equation for incompressible flow and later introduced compressibility via the continuity equation, keeping acceleration terms in the process. Their entire analysis is predicated on the smallness of pore thickness and on the importance of fluid viscosity. These two properties are parameterised by the Reynolds numbers, which are infinitesimal for conditions of interest. Under these circumstances, neglect of compressibility in the momentum equation seems justified by Eq-3.33, as is accounting for it in the continuity equation where compressibility is first order (Eq-3.34). However, keeping acceleration terms is most inconsistent with respect to the rigorous perturbation analysis presented in section 3.2.3, which indicates that *all* first order inertial terms are weighted by the Reynolds numbers. Such inconsistency does not invalidate Murphy & al.'s final results because inertial effects were set to zero in the solutions found for Eq-3.58.

Solutions to Eq-3.64 can be expected to be both simpler and physically consistent.

More problematic is the squirt-flow model of Mavko & Nur (1979) in which acceleration effects are central. Their governing equation is expressed in terms of velocity and so is not amenable to direct comparison with my pressure-field equations⁴; however, in a recent contribution (Dvorkin, Mavko & Nur, 1990) they extended their model by calculating the pressure-field inside an oscillating fracture filled with a compressible fluid, using the same premises than here. Their equation (10), for one-dimensional flow within non-parallel, impermeable walls reads

$$\left(h - \sqrt{\frac{\nu}{i\omega}} \tanh \left[h \sqrt{\frac{i\omega}{\nu}} \right] \right) \frac{d^2 p}{dx^2} + \left(\frac{\partial h}{\partial x} \tanh^2 \left[h \sqrt{\frac{i\omega}{\nu}} \right] \right) \frac{dp}{dx} + \left(\frac{\omega^2 h}{C^2} \right) p = -\rho_0 \omega^2 \xi h_0 \quad [3.65]$$

where $h = h(x)$; $\nu = \mu / \rho_0$ (kinematic viscosity); and $C = [1 / \rho_0 \beta]^{1/2}$ (sound velocity in the fluid, taken as a measure of compressibility).

This problem can be expressed within the framework of squeeze-flow equation, Eq-3.53, using a linearisation procedure similar to Eq-3.59 — Eq-3.61 but for cartesian coordinates. Making the required substitutions, I obtain

$$\{h^3\} \frac{d^2 p}{dx^2} + \{3h^2 \frac{\partial h}{\partial x}\} \frac{dp}{dx} - \{i\omega 12\mu h \beta\} p = i\omega 12\mu h_0 \xi \quad [3.66]$$

which, when multiplied by $[i\omega \rho_0 / 12\mu]$, yields

$$\left\{ \frac{i\omega}{12\nu} h^3 \right\} \frac{d^2 p}{dx^2} + \left\{ \frac{i\omega}{4\nu} h^2 \frac{\partial h}{\partial x} \right\} \frac{dp}{dx} + \left(\frac{\omega^2 h}{C^2} \right) p = -\rho_0 \omega^2 \xi h_0 \quad [3.67]$$

The last two terms on the right of this equation are identical to that of Eq-3.65; however, the coefficients of the first two, whilst functions of $h(x)$, ω and ν , differ because acceleration terms are absent from squeeze-flow. Not surprisingly, these coefficients are simpler than those corresponding to Dvorkin & al's model whose results come into question. However, their interests are not limited to seismic frequencies and thin fractures; for frequencies in the kHz range and beyond, Reynolds number R_s may become larger than one in which case Dvorkin & al are justified to include at least acceleration terms and perhaps should include convective terms as well, particularly for thick fractures. At any rate, however, taking the limit as $\omega \rightarrow 0$ of the first two coefficients of Eq-3.65 I find that both monotonically approach zero as do the corresponding coefficients of Eq-3.67; therefore, Eq-3.67 can be viewed as the low-frequency limit of Eq-3.65.

⁴ Mavko & Nur (1979) governing equation can be deduced from Eq-3.33, by setting $\bar{p} = 1$ (incompressible) and by neglecting all inertial terms, except acceleration (weighted by R_s).

Synopsis of thin-film flow governing equations

general fluid-film Eq-3.41 (this work)

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial(\bar{\rho}H)}{\partial T} + \frac{6}{\omega S} \frac{\partial}{\partial X_k} \left[\bar{\rho}H (U_a + U_b + U_k) \right] - \frac{\partial}{\partial X_k} \left(\bar{\rho}H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s)$$

gas-film Eq-3.44 (Langlois, 1962)

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial}{\partial T} (\bar{P}H) + \frac{6}{\omega S} \frac{\partial}{\partial X_k} [\bar{P}H (U_a + U_b + U_k)]$$

$$- \frac{\partial}{\partial X_k} \left(\bar{P}H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s)$$

liquid-film Eq-3.45 (this work)

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial}{\partial T} (He^{c\beta}(\bar{P})) + \frac{6}{\omega S} \frac{\partial}{\partial X_k} [He^{c\beta}(\bar{P}) (U_a + U_b + U_k)]$$

$$- \frac{\partial}{\partial X_k} \left(e^{c\beta}(\bar{P}) H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s)$$

Low-Reynolds numbers (lubrication) Eq-3.47

$$\frac{\partial}{\partial X_k} \left(\bar{\rho}H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = \sigma \frac{\partial}{\partial T} (\bar{\rho}H) + \Lambda \frac{\partial}{\partial X_k} [\bar{\rho}H (U_a + U_b + U_k)]$$

O'Connell & Budiansky (1977)
[incompressible fluid $\beta = 0$]

generalised squeeze-flow Eq-3.53

Murphy & al. (1986); $\beta \neq 0$

$$\frac{\partial}{\partial X_k} \left(\bar{\rho}H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = \sigma \frac{\partial}{\partial T} (\bar{\rho}H)$$

Low-frequency limit

Mavko & Nur (1979); $\beta = 0$
Dvorkin & al. (1992); $\beta \neq 0$

Figure-3.6 Summary of the thin film flow equations

Given some clarifications about the physics of constricted thin-films, the purpose of this chapter was to develop — from a minimum of assumptions — a mathematically and physically consistent theory of fluid-film motion within the fundamental site of deformational energy dissipation in saturated, cracked rock: a sinusoidally stressed pore of low aspect-ratio (i.e. $10^{-6} < \epsilon < 10^{-2}$). Premises of the theory amount to four conditions on fluid-film properties, whether gas or liquid: 1) Newtonian; 2) isoviscous; 3) adherent; and 4) pressure-dependent density, when compressible. A dimensionless governing equation (Eq-3.41) — correct to first order in ϵ — was first obtained for the pressure-flow field generated in a fluid-film by two bounding solid-surfaces of arbitrary shape and mobility. Prejudice about the nature of flow regime was avoided by retaining all non-linear and inertial terms, expressing them in an integro-differential form. In that sense, Eq-3.41 is general. Explicit governing equations were then found for gases (Eq-3.44) and liquids (Eq-3.45) obeying, respectively, the perfect gas law and an equation of state with density an exponential function of pressure, weighed by compressibility (Eq-3.1). Figure-3.6 summarises the various governing equations encountered in lubrication and in "squirt-flow" modelling which can be deduced from the general equation, Eq-3.41.

Of particular interest to seismic dissipation is the generalised squeeze-flow equation Eq-3.53 (& Eq-3.55) which, together with Reynolds number R_s and squeeze-number σ , comprise — succinctly — the relevant quantities of intrinsic attenuation by viscous losses in a compressible fluid. Both R_s and σ encompass important characteristics of crustal fluids: density, viscosity, thickness, width, nominal pressure, and frequency of excitation. Interestingly, despite being a clear discriminant of flow regime, Reynolds number R_s has been ignored in squirt-flow models, resulting either in over-parameterisation (Murphy & al., 1986) or possible under-parameterisation (Dvorkin & al., 1992). As they stand, the applicability of these last two models to crustal seismics seems uncertain: however, the above formalism allows assessment of the numerous assumptions used in their development, for a large number of situations.

Using the theory described here, it is now possible to objectively estimate the importance of dynamic terms affecting fluid-film flow of seismological interest, prior to attempt solving a problem. Whilst conventional treatments of thin-film hydrodynamics avoid the rigour of the theory developed here, there are advantages in having the complexities of confined fluid flow made mathematically explicit, particularly in dimensionless form. Apart from providing strength to otherwise questionable arguments, theoretical results (e. g. Eq-3.35, Eq-3.53, Eq-3.55) suggest ways of approaching problems of growing interest. For example, the problem of non-linear attenuation, of great importance for seismic source studies, could be approached by exploring numerical solutions to the non-linear partial differential equation, Eq-3.53, for large pore-wall motion. It may also be possible to directly solve the Fredholm equation, Eq-3.35, using recent techniques proposed by Press & Teudolsky (1990). However, such special problems are not considered in this thesis: the full implications of the linearised governing equation,

still not being well-understood, remain the primary target of this work.

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Chapter-4: Seismic dissipation by thin-film squeeze-flow: two models for crustal fractures¹

In this chapter I apply the theory developed in chapter-3 to water-saturated crustal cracks whose dimensions are inferred from rock-surface roughness measurements and from manifestations of crustal hydraulic properties. It is important to recognise that these dimensional constraints are field-based, not laboratory-based. Here, the purpose is to establish two main features of the physics of seismic dissipation by compressible squeeze-flow, for crustal conditions: 1) a characteristic frequency of maximal dissipation which, as depth increases, remains within the crustal seismic band [.08 to 25 Hz]; and 2) a magnitude of dissipation which is significant within this band and depth range. These two features are required if a depth distribution of intrinsic attenuation [i-e $Q_i^{-1}(\omega)$] is to explain a significant portion of the frequency-dependent attenuation observed over a broad bandwidth at the surface of the crust [i-e $Q_t^{-1}(\omega)$].

The peculiar geometry of crustal cracks imposes specific boundary conditions which may be unique to fractured rock; consequently, studying fluid-flow in such cracks raises unusual problems of viscous hydrodynamics in constricted space. Here, I solve — analytically — two such problems in terms of crack stiffness using idealised boundary conditions for a single characteristic crack, arguably the basic element of mechanical energy dissipation in a saturated, fractured medium. Solutions to these single-crack problems, together with constraints on porosity of fractured formations, can later be cast in an effective-medium scheme for calculating the crustal distribution of seismic dissipation proper — $Q_i^{-1}(\omega)$ and associated dispersion $v(\omega)$. This is done in the next chapter. Here, only that part of crack-stiffness which contributes to dissipation is calculated, namely that influenced by pore-fluid behaviour. The elastic deformation associated with solid-solid contacts of crack surfaces will be accounted for in the computation of the effective-medium response.

In section 4.1, I examine the squeeze-flow phenomenon in a conceptual crustal environment, and present estimates of two informative dimensionless groupings introduced in chapter-3: Reynolds number R_s , which constrains the regime of oscillatory fluid-flow; and squeeze-number σ , which underlines the importance of fluid compressibility. From these estimates, the validity

of the generalised squeeze-flow governing equation — Eq-3.53 — is established for seismically-stressed crustal cracks. In section 4.2, the micro-geometry representation of a characteristic crack is modelled together with two types of boundary conditions appropriate for the intricate interspace of rock fractures. A reduced, linear governing equation of squeeze-flow for such representation and boundary conditions is then solved for saturated crack stiffness as a function of frequency, thereby illustrating the physics of viscous dissipation operative in the crustal phase band. In section 4.3, results of the two models of crustal cracks are compared and their sensitivity to modelling parameters is discussed.

4.1 Model description

Figure-4.1 illustrates the essence of my approach. Crustal formations are regarded as an arrangement of macroscopic blocks, each separated by corrugated fractures. The basic postulate is that mechanical energy transported by seismic waves is preferentially absorbed by viscous flow of sinusoidally-squeezed fluid-films, trapped in the tortuous fracture interspace. It is assumed that the macroscopic blocks themselves are perfectly elastic and that absorption mainly occurs at block contacts, within microscopic sites. Viewed in cross-section, a typical dissipative site has two structural components: 1) two roughly parallel plates of

Conceptual Model of Crustal Rock with Dissipative Fractures

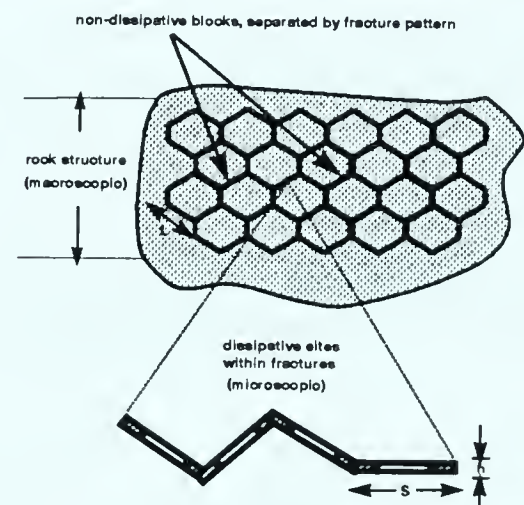


Figure-4.1 Cross-section of a crustal-scale rock formation, viewed as a packing of macroscopic blocks whose contacts are corrugated at the microscopic scale. Dissipation of seismic energy results from viscous flow of interstitial fluid, within the interspace of fractures. Dissipative sites consist mainly of a narrow fluid-saturated gap surrounded by porous material. Site orientations are supposed random, in all three spatial coordinates. e.g. orders of magnitude: $L \sim O(10^2 \text{ m})$; $h_0 \sim O(10^{-8} \text{ m})$; $S \sim O(10^{-3} \text{ m})$.

impervious material, separated by a fluid-film; and 2) a peripheral porous medium of finite extent, whose average pore-pressure differs from that of the squeezed-film. The first component, in which most of the fluid lies, embodies the viscous dissipation process. The second component, having less fluid per unit volume, primarily plays a role

¹. An earlier version of this paper was presented at the Canadian Geophysical Union 17th annual meeting in Banff, Alberta, on May 8th, 1991 (paper # 38, Rouleau, P.M. A model for hydrodynamic relaxation in crustal rocks: implications for seismic wave attenuation).

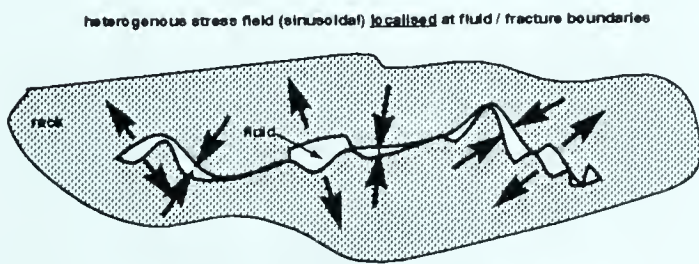
of moderator of viscous fluid-flow. I shall refer to the parallel-plate arrangement as the gap and to the bounding medium as the ring. The combination gap-ring will be called the dissipative cell.

4.1.1 Dissipation mechanism in fractures

The source of dissipation by local fluid-flow in granular rocks (alias "squirt-flow") is well-known: the process is driven by the highly heterogeneous stress-field arising from the distribution of normal and tangential stress components around each grain surfaces [for a succinct summary, see O'Connell, 1984]. Because many communicating cracks present different orientations to the main stress-field, heterogeneous stresses generate local pressure gradients which then impart fluid motion; therefore, viscous losses. Within the conceptual framework of fig-4.1, this mechanism, here called squeeze-flow, is equally applicable. Indeed, as illustrated in fig-4.2, the likelihood of a localised stress-field conducive to viscous flow is high when considering that rough-walled fractures, whilst appearing as two-dimensional structures at the macroscopic scale, are in fact three-dimensional bodies at a much smaller scale.

Roughness of natural fractures is such that the topography of their walls is often fractal and so devoid of a characteristic dimension; yet, when two contacting such walls form an open fracture at depth, a characteristic pore-length of low aspect-ratio does exist because only longer topographic wavelengths compose contacting surfaces. Furthermore, pore-length scales with corresponding height from characteristic length downward [cf. chapter-2]. These remarkable features were pointed-out by Scholz [1988] in the context of a fracture instability study, in which the smallest length asperity, noted λ_c , was found to be both an instability controlling parameter and the most common size in a distribution of asperity

Source of Seismic Dissipation



mechanism of absorption: squeeze-flow of viscous thin-films

Figure-4.2 Source of motion of viscous fluid within fractured rock. General stress-field $\tau_{i,j}$ associated with a passing seismic wave of frequency ω , generates normal components on dissipative site walls and so pressurises interstitial fluids by an amount $p(\omega) = -\tau_{i,j} n_i n_j$ where n_i, n_j are unit normals to site walls. Arrows indicate orientations of localised, oscillatory stress-components. Fracture size $\ll$ seismic wavelength.

lengths. On the geometrical implication of his work, Scholz stated: "... in particular, λ_c is the maximum con-

tact spacing and it is also the minimum diameter of mated junctions...". This result provides a rule of thumb for visualising fracture interspace and so some suggestions for modelling fluid-flow therefrom.

The application to squeeze-flow is that λ_c constitutes a characteristic dimension of the largest and most common space available to fluids inside a fracture, here taken as a thin network of randomly oriented dissipative cells. The rule also constrains the connectivity of those cracks which are characteristic. Although the distance between individual characteristic cracks can vary widely, the proximity of any two such cracks is at most λ_c (adjacent cracks closer than λ_c are perforce smaller and so not characteristic). Based on these geometrical constraints, I shall assume that the gap of a dissipative cell approximates a thin disk of diameter $\sim \lambda_c$ and that the peripheral ring is a porous mesh of smaller voids, with its minimal length $\sim \lambda_c$. Although this imposes some order to an otherwise disordered geometry, all opened spaces in a distribution of void sizes enter in the squeeze-flow modelling, albeit in an idealistic way. In return for such idealisation, however, the squeeze-flow problem remains mathematically tractable.

Moreover, I shall assume that viscous squeeze-flow is confined to single cells and so keep in line with squirt-flow results which have shown that wave-energy attenuation and dispersion in permeable rock-samples is most likely a local (pore-scale) process, not one arising from macroscopic fluid-rock interactions [e. g. Murphy et al., 1986].

4.1.2 Constraints on flow nature: Reynolds and Squeeze numbers

Dissipation by squeeze-flow is more conveniently modelled in terms of the pressure-field generated in the pore-fluid by stresses acting on confining surfaces. As I have shown in section-3.2.4, the general pressure-flow of a viscous fluid in an oscillating crack of low aspect-ratio and arbitrary topography is governed by the reduced Navier-Stokes equation (Eq-3.41):

$$\frac{12}{(1 + \frac{V}{\omega S})} \frac{\partial(\bar{\rho}H)}{\partial T} + \frac{6}{(1 + \frac{\omega S}{V})} \frac{\partial}{\partial X_k} [\bar{\rho}H ({}_a U_k + {}_b U_k)] - \frac{\partial}{\partial X_k} \left(\bar{\rho}H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_l I_l + R_s (I_l + I_s) \quad [4.1]$$

where all symbols are dimensionless and have the same meaning as before. Here, I justify further reduction of this governing equation for squeeze-flow in a dissipative cell of the type described above.

Both terms on the right-hand side of Eq-4.1 account for inertia, with Reynolds numbers R_s (squeezing mode) and R_l (lateral mode) reflecting their magnitudes. When lateral motion of confining surfaces is negligible relative to squeezing, as is the case for seismically-stressed crustal cracks [cf. section-3.3.1], $R_s \gg R_l$ and ${}_a U_k = {}_b U_k = V \sim 0$. Under such conditions Eq-4.1 becomes

$$12 \frac{\partial(\bar{\rho}H)}{\partial T} - \frac{\partial}{\partial X_k} \left(\bar{\rho} H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = R_s (I_l + I_s) \quad [4.2]$$

This equation serves as a basis for assessing inertial flow in crustal cracks, together with the definition of R_s given by (Eq-3.30):

$$R_s = \frac{\rho_0 h_0^2 \omega}{\mu} \quad [4.3]$$

Figure-4.3 shows a log-log plot of R_s as a function of frequency ω for density ρ_0 and viscosity μ proper to water, with crack thickness h_0 as parameter. Clearly, for all but the largest thicknesses ($h_0 > 10^{-3}$ m), $R_s \ll 1$ throughout the crustal phase band. Although very thick cracks certainly exist in the uppermost layers of the crust, they are not representative of rock formations at the crustal scale. Furthermore, characteristic pore-length λ_c for clean crustal fractures is $\sim 10^{-3}$ m to which corresponds a thickness $h_0 \sim 10^{-7}$ m; therefore, inertia is quite negligible in the squeeze-flow regime of interest here.

Having established this, the right-hand side of Eq-4.2 can be set to zero, in which case the pressure-flow in the crack is governed by the generalised squeeze-flow relation (Eq-3.53):

$$\frac{\partial}{\partial X_k} \left(\bar{\rho} H^3 \frac{\partial \bar{P}}{\partial X_k} \right) = \sigma \frac{\partial}{\partial T} (\bar{\rho} H) \quad [4.4]$$

where all symbols are dimensionless and as defined in chapter-3, with pressure normalised to ambient fluid pressure p_0 . The question now is whether fluid

little help for a straightforward answer, however, squeeze number σ can be used for determining if fluid compression is effected, by first treating it as a perturbation parameter. Assuming small excess pressure $\bar{p}$ about ambient for the condition $\sigma < 1$ [e. g. at very low frequency], $\bar{p}$ can be expanded accordingly to

$$\bar{p} = 1 + \sigma \bar{p}_1 + O[(\sigma \bar{p}_1)^2] \quad [4.5]$$

Inserting this expansion in Eq-4.4 and retaining only first order terms yields

$$\frac{\partial}{\partial X_k} \left(H^3 \frac{\partial \bar{p}_1}{\partial X_k} \right) = \sigma \frac{\partial H}{\partial T} \quad [4.6]$$

where density no longer appears and so demonstrates that at low squeeze number flow is essentially incompressible. Therefore, the magnitude of σ attests of the importance of fluid compressibility, the justifying criterion being $\sigma \geq 1$.

The defining expression for σ reads (Eq-3.50)

$$\sigma = \frac{12\mu\omega}{p_0} \frac{1}{\epsilon^2} \quad [4.7]$$

where ϵ is crack aspect-ratio. Eq-4.7 shows that σ is proportional to the product $\mu\omega$ and scales inversely with p_0 and ϵ^2 . For water-filled characteristic cracks at depth 15 km, $\epsilon \sim 10^{-5}$, $p_0 \sim 150$ MPa (hydrostatic), $\mu \sim 10^{-3}$ Pa-s, giving $\sigma \sim 0.8 \omega$. Thus, even near one Hz σ is of order unity. For $\epsilon \geq 10^{-4}$, other parameters same, $\sigma \leq 0.008 \omega$, suggesting that squeeze-flow is incompressible in thicker cracks. For $\epsilon \sim 10^{-6}$, the smallest acceptable aspect-ratio, $\sigma \sim 80 \omega$, underlining the importance of fluid compressibility. Interestingly, if ambient pressure approaches lithostatic, for instance in saturated formations trapped at depth, cracks with lowest ϵ still yield $3 \omega \leq \sigma \leq 30 \omega$, depending on viscosity (itself function of temperature).

I estimated the magnitude range of squeeze number σ as a function of crustal depth z_c , using frequency ω as parameter and taking ambient pressure p_0 proportional to z_c with a pressure-gradient of 15 Mpa/km (midway between hydrostatic and lithostatic). ϵ was fixed at 10^{-5} . The result, shown in fig-4.4, indicates that for crustal seismic frequencies $\sigma \geq 1$ throughout the upper crust, thereby demonstrating the importance of taking fluid compressibility into account. This conclusion is reinforced for characteristic cracks of aspect-ratio nearing 10^{-6} . These considerations fully justify application of the general squeeze-flow governing equation (Eq-4.4) to characteristic crustal cracks subjected to localised seismic stress.

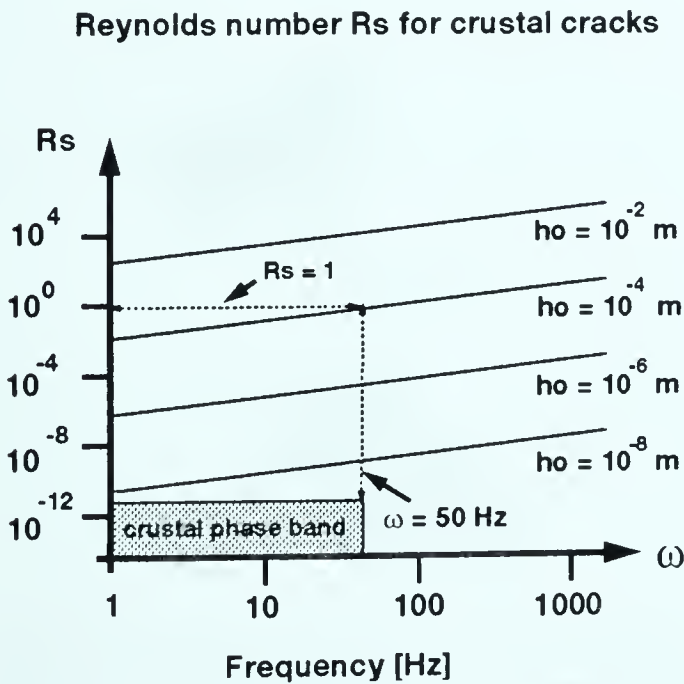


Figure-4.3 Range of Reynolds number R_s for water-saturated cracks at crustal conditions, as a function of frequency of squeezing [Eq-4.3]. Parameter h_0 is crack thickness. Shaded box shows the crustal seismic band-width.

compressibility is significant for squeeze-flow in crustal cracks, at seismic frequencies. As it stands, Eq-4.4 offers

Squeeze number σ for crustal cracks

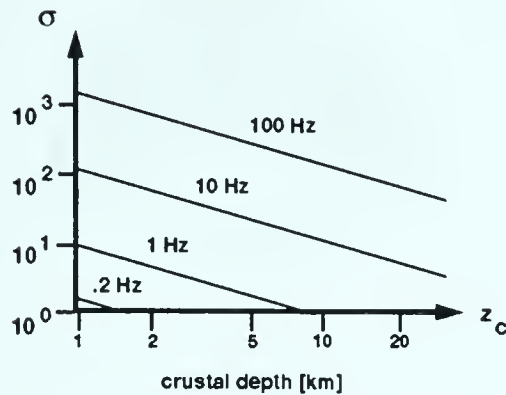


Figure-4.4 Magnitude range of squeeze-number σ for seismically stressed crustal cracks. Depth dependency follows from setting fluid ambient pressure p_0 as a linear function of depth z_c , with a gradient of 15 MPa/km. Characteristic crack aspect-ratio $\epsilon = 10^{-5}$. Fluid viscosity $\mu = 10^{-3}$ Pa-s (water). Frequency parameter ω covers crustal seismic bandwidth.

4.2 Squeeze-flow in a characteristic dissipative cell

Figure-4.5 illustrates how any two characteristic crustal cracks may look inside a natural fracture. They have separation $\geq \lambda_c$ and low aspect-ratio $\sim h_0/\lambda_c$. The upper and lower surfaces of each are joined at discrete points by deformable solid material, in between which a saturating fluid can flow. Porous matter between

Two characteristic cracks in a rough fracture

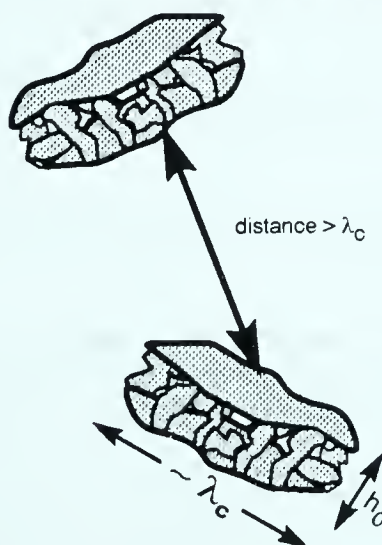


Figure-4.5 Model of characteristic cracks within the interspace of a natural fracture. λ_c is both characteristic crack-length and minimal distance between two such cracks. h_0 is crack thickness. Crack periphery is regarded as the entry of a porous mesh through which a viscous fluid can flow.

characteristic cracks (not shown) can be viewed as a complicated mesh of conduits, whose variable diameters

equal joint separations at crack peripheries and whose orientations differ from that of the cracks, and from each other. Such configuration is clearly propitious to fluid exchanges by squeeze-flow between a crack and its immediate vicinity.

Figure-4.6 represents an idealisation of this situation — a dissipative cell. Crack surfaces are coaxial disks with parallel walls, nominally separated by h_0 and remaining parallel under load. The gap structure is in radial contact with a permeable ring of finite extent. As the gap responds to sinusoidal stress, the bounding ring structure may or may not deform depending on the heterogeneous stress-field acting on it². This raises two possibilities for the role of the boundary pore-fluid in the squeeze-flow process operating in the gap. If the heterogeneous stress is such that ring conduits negligibly deform and pressure remains ambient then the ring can be assumed rigid, in which case gap-flow will primarily be moderated by fluid compressibility and by the simple fluid-volume capacity of ring pores. If, however, the heterogeneous stress is efficient at deforming ring conduits (logically, it is likely) then more than pore-fluid compressibility and ring porosity will moderate flow in and out of the gap, particularly if the ring effectively dilates as the gap compresses. Hereon, the first situation will be referred to as the passive ring problem, the second as the active ring problem.

4.2.1 Reduced governing equation for dissipative gap

The central dissipative element of both active and passive ring problems remains the saturated gap in which squeeze-flow is governed by Eq-4.4. Since the dissipative cell has a disk shape, it is best to work with cylindrical coordinates for which Eq-4.4 reads

$$\frac{\partial}{\partial R} (R \bar{\rho} H^3 \frac{\partial \bar{p}}{\partial R}) = R \sigma \frac{\partial}{\partial T} (\bar{\rho} H) \quad [4.8]$$

(cf Eq 3.55) where it is understood that fluid flow is averaged over thickness. For fluid in the liquid state, density function $\bar{\rho}$ can be related to pressure by the equation of state

$\bar{\rho} = e^{c \bar{p}}$, with c comprising liquid compressibility β , as discussed in chapter-3.

². At the gap-ring junction solid surfaces contacts obviously deform in phase with characteristic crack walls. Such “end effects” are not considered here, the overall cell response being the main concern.

Idealised dissipative cell

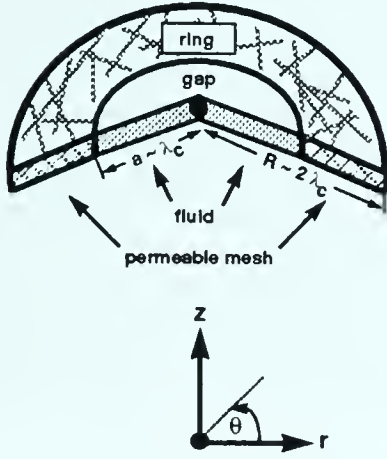


Figure-4.6 Perspective view of an idealised cell, taken as a representation of a characteristic crack. The gap consists of two parallel disks of radius $a \sim \lambda_c$, separated by a thin fluid of nominal thickness h_0 . The ring is modeled as a permeable mesh of randomly oriented fluid conduits, with radius $R \sim 2 \lambda_c$. Bottom shows the radial coordinate system used in modeling squeeze-flow in the cell.

Equation-4.8 is highly non-linear; however, because of the smallness of seismic stress away from the source, a linearisation is quite justified. Also, the parallelism of gap walls renders H independent of radius. As shown in section-3.3.2, assuming H varies harmonically by a small amount ξ about its mean, and expanding the associated pressure $\bar{p}$ about ξ and $\bar{\rho}$ about compressibility β , the first order form of Eq-4.8 becomes the linear steady-state equation for pressure $\bar{P}_1$ (Eq-3.63)

$$\frac{\partial^2 \bar{P}_1}{\partial R^2} + \frac{1}{R} \frac{\partial \bar{P}_1}{\partial R} - i \frac{\sigma}{H^3} \beta p_0 \bar{P}_1 = i \frac{\sigma}{H^3} \xi \quad [4.9]$$

In terms of the coordinate system of fig-4.6, the dimensional form of this equation is

$$\frac{\partial^2 p_g}{\partial r^2} + \frac{1}{r} \frac{\partial p_g}{\partial r} - \beta h_0 \left(i \omega \frac{12\mu}{h_0^3} \right) p_g = i \omega \frac{12\mu}{h_0^3} \zeta h_0 \quad [4.10]$$

Eq-4.10 is an inhomogeneous, parabolic, linear differential equation of second order, which governs the vertically-integrated, radial (excess) pressure p_g in a viscous liquid-film of a gap being squeezed at frequency ω by an amount ζh_0 . It is to be solved subject to boundary conditions appropriate for the active and passive ring configuration. The type of this differential equation becomes apparent when defining parameter κ as

$$\kappa = \sqrt{-i} \sqrt{\frac{12\mu\beta}{h_0^2} \omega} \quad [4.11]$$

with which Eq-4.10 reads

$$\frac{\partial^2 p_g}{\partial r^2} + \frac{1}{r} \frac{\partial p_g}{\partial r} + \kappa^2 p_g = i \omega \frac{12\mu}{h_0^3} \zeta h_0 \quad [4.12]$$

Eq-4.12 reveals a Bessel equation of order zero with source term on the right, for which the general solution is well-known. Here, the general solution to Eq-4.12 can be written as

$$p_g(r, \omega) = c_0 + c_1 J_0(\kappa r) + c_2 Y_0(\kappa r) \quad [4.13]$$

where J_0 and Y_0 are, respectively, Bessel functions of the first and second kind, with complex argument; constant c_0 relates to the source term, and constants c_1 and c_2 are determined by appropriate boundary conditions.

At the gap-ring junctions ($r = a$), there are two boundary conditions common to both active and passive ring problems: equality of pressures; and equality of flow-rates. With p_g referring to gap-fluid pressure and p_r to ring-fluid pressure, the first is simply written

$$p_g(a, \omega) = p_r(a, \omega) \quad [4.14]$$

The second can be stated in the time domain in terms of flow-rate $q_{g,r}(t)$ [in units of m^3/sec] as

$$q_g(t) + V_g \beta \frac{\partial p_g}{\partial t} = q_{ring}(t) \quad [4.15]$$

where V_g is the gap nominal volume. The first term on the left-hand side of Eq-4.15 reflects the rate of gap-volume change during thickness variation, the second the rate of compressible gap-fluid volume during pressure variation. The term on the right is the balancing flow rate in and out of the ring. For a squeezing disk-shape gap of nominal thickness h_0 Eq-4.15 reads

$$\pi a^2 \frac{\partial h}{\partial t} + \pi a^2 h_0 \beta \frac{\partial p_g}{\partial t} = q_{ring}(t) \quad [4.16]$$

The frequency-domain form of this equation is obtained by taking $h = h_0 (1 + \zeta e^{i\omega t})$, $\zeta \ll 1$; $P_g = p_g e^{i\omega t}$, and $q_{ring} = q_r e^{i\omega t}$, which gives

$$i \omega [\pi a^2 \zeta + \pi a^2 h_0 \beta p_g(a, \omega)] = q_r(a, \omega) \quad [4.17]$$

The explicit expression for $q_r(a, \omega)$ depends on the fluid-flow model for the ring. I shall treat the easier problem of a dissipative cell with rigid (passive) ring first. The more complicated problem of a cell with active ring requires establishing an additional governing equation for pressure-flow in a deformable permeable medium and so requires more boundary conditions.

4.2.2 Solution to passive ring problem

When the bounding ring is effectively rigid, fluid-flow in and out of the squeezing gap is moderated by the elas-

ticity of pore-fluid residing in the permeable mesh at ambient pressure. An electrical analogue is useful in portraying the process. Under gap compression, the volume of residing pore-fluid can be viewed as a capacitor which, given enough time, will accumulate a net charge. Thus, a fluid-storing capacity can be defined. Murphy et al. (1986), in their squirt-flow model of a squeezing pair of grain surfaces in contact with a large toroidal fluid-reservoir, defined such a capacity as the volume of the reservoir divided by fluid bulk modulus. Of course, such a large reservoir cannot exist in the interspace of rough fractures without violating roughness constraints; however, it is reasonable to regard a gap-bounding ring as a torus of equal volume, whose pore-fluid occupies a fraction of it given by ring porosity. In such case, the fluid-storing capacity of a cell ring can be defined as

$$C = 2\pi^2 (a+b) b^2 \cdot \phi\beta \quad [4.18]$$

where β is fluid compressibility, ϕ porosity, and their common factor, the volume of a torus of radius b , with a the radius of the gap. C has units $[m^3/Pa]$ and so reflects the ring capacity of accepting fluid per unit rise in pressure.

Solving governing equation Eq-4.12 for the passive ring cell requires two boundary conditions for determining c_1 and c_2 in the general solution Eq-4.13. Since gap-fluid pressure is at all times finite at the origin,

$$|p_g(0, \omega)| < \infty \quad [4.19]$$

Recollecting that Bessel function Y_0 is infinite at the origin, Eq-4.13 and Eq-4.19 imply $c_2 = 0$; therefore, the general solution to Eq-4.12 becomes

$$p_g(r, \omega) = c_0 + c_1 J_0(\kappa r) \quad [4.20]$$

Constant c_0 is found by simply inserting this equation into Eq-4.12, following which

$$c_0 = -\frac{\zeta}{\beta} \quad [4.21]$$

The remaining constant c_1 is determined by flow-rate equality at the gap-ring junction, for which the condition of Eq-4.16 reduces to

$$\pi a^2 \zeta h_0 + \pi a^2 h_0 \beta p_g(a, \omega) = C p_g(a, \omega) \quad [4.22]$$

where use has been made of Eq-4.14. Similarly to Eq-4.15, the first term on the left-hand side of Eq-4.22 expresses the variation of the gap volume due to departure ζh_0 from nominal thickness h_0 , the second the variation of the compressible gap-fluid volume due to excess pressure. The term on the right expresses fluid volume exchanges accordingly to the ring storing capacity, for excess pressure p_g at $r=a$.

For fully exploiting recurrence properties of Bessel functions, it is useful to re-write Eq-4.22 as

$$\pi a^2 \zeta h_0 + \frac{2\pi\beta h_0}{a} \int_0^a p_g(r, \omega) r dr = C p_g(a, \omega) \quad [4.23]$$

where the integral term comes from using the definition of pressure p_g as the load F (force) acting on disk surfaces divided by their area (πa^2), with F given by

$$F = 2\pi \int_0^a p_g(r, \omega) r dr \quad [4.24]$$

This introduces Bessel functions of higher order in the solution, which then can be easily calculated.

Inserting Eq-4.21 and Eq-4.23 in Eq-4.20 and then setting $r=a$ solves for c_1 , which then leads to the solution

$$p_g(r, \omega) = -\frac{\zeta}{\beta} \cdot \left[1 - \frac{2\Theta_p J_0(\kappa r)}{2\pi a^2 h_0 J_1(\kappa a) + \kappa a \Theta_p J_0(\kappa a)} \right] \quad [4.25]$$

where $\Theta_p = C/\beta$, a constant; and $J_1(\kappa a)$ is Bessel function of order one related to $J_0(\kappa r)$ by

$$J_1(\kappa a) = \frac{1}{a} \cdot \int_0^a J_0(\kappa r) \kappa r dr \quad [4.26]$$

The complex pressure in Eq-4.25 has real and imaginary parts whose associated phase lag can be related to dissipation, as discussed in section-4.3.

4.2.3 Solution to active ring problem

For a dissipative cell with active ring, fluid-flow in its gap is still governed by Eq-4.12, and the condition of finite pressure at the origin (Eq-4.19) still holds; therefore, pressure-flow of gap-fluid is again given by

$$p_g(r, \omega) = c_0 + c_1 J_0(\kappa r) \quad [4.27]$$

with c_0 as in Eq-4.21. As before, c_1 is constrained by continuity of pressure and flow-rate at the gap-ring junction, $r=a$.

However, the problem here addresses the likely situation that the ring pore-structure is such that the unit normal of most of its voids differs from that of gap surfaces and from one void to the other. In such configuration, the heterogeneous stress-field localising onto the cell logically generates a pressure gradient between gap and ring as well as through the ring itself. This suggests that the ring can be regarded as though it dilates during gap compression and vice-versa, at frequency ω . The mechanical analogue shown in figure-4.7 helps in visualising the process.

Mechanical analogue of squeeze-flow in cell with active ring

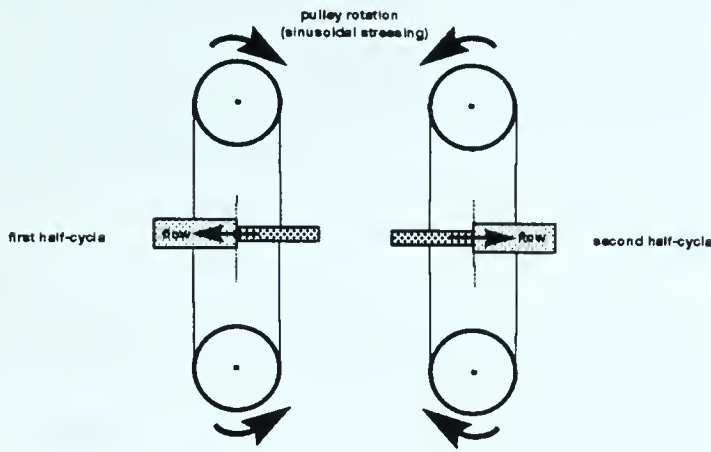


Figure-4.7 Analogue of oscillatory squeeze-flow in a cell with active ring. Sinusoidal rotation of pulleys mimics heterogeneous stress components acting simultaneously on gap (left arm) and ring (right arm) surfaces. Viscous fluid flows back and forth the gap-ring space, accordingly to cycling frequency ω .

Confining surfaces of the ring as a whole need not be set into motion to capture the dynamic character of this process because some conduits in the mesh logically dilate while others compress; rather, the important dynamical element is the ensuing pressure-gradient across the ring. Assuming stationary ring walls avoids the need for an additional driving term, the source of excess pressure being that emanating from the gap at the ring entry. Because of this and because the ring is a deforming permeable structure quite unlike a simple parallel-plate gap, flow of ring-fluid obeys a governing equation different from Eq-4.12. Under such condition, an effective strategy for attacking the active ring problem is to establish the ring-fluid governing equation and then solve it for pressure. This allows defining the conditions of pressure and flow-rate continuity at the gap-ring junction in the same terms than Eq-4.14 and Eq-4.16. In particular, $q_r(r, \omega)$ can be expressed using the well-known Darcy's law.

Governing equations for oscillating porous media are usually expressed in terms of velocity of solid and fluid parts, their exact form being a matter of recurring debates. Common practise consists in employing Biot's theory of wave-propagation in saturated poroelastic media, which predicts a bell-shaped frequency-dependent attenuation (Biot, 1956). However, the phenomenological premiss of this theory can only lead to maximal dissipation by interaction of viscous and inertial processes and the latter is not expected to operate at seismic frequencies in pores of characteristic crack attributes.

Here, I adopt a governing equation recently presented by Reh binder (1989) for flow of a compressible interstitial liquid in a deforming permeable medium³. His contribution, which is both theoretical and experimental, is

³. Apparently unbeknownst to Reh binder, his theoretical results were found much earlier by Oroveanu and Pascal (1959) and Pascal (1969) to whom full credit must be attributed. Reh binder's equations are used here because of notation similarities.

quite appropriate not only because it gives a governing equation in terms of liquid-pressure but also because it addresses inertia effects in a physically transparent manner, thereby allowing assessment of their importance for pores unlike characteristic cracks. Moreover, due attention is paid to the time-variation of fluid density, matrix porosity and permeability associated with a small pressure disturbance in the medium. Reh binder's (1989) result is based on a momentum equation formed by adding an inertial term to Darcy's law and on a dimensional analysis of the terms which arise when velocity is eliminated from momentum and continuity equations, density being an exponential function of pressure (as in Eq-3.1). Although this approach is semi-empirical, it leads to results which are consistent with observation, particularly when the medium is subjected to a small harmonic perturbation [Reh binder, 1992].

Reh binder's (1989) governing equation for pressure-flow P_r of a slightly perturbed compressible pore-liquid reads

$$\frac{k\rho_0}{\mu\phi} \frac{\partial^2 P_r}{\partial t^2} + \frac{\partial P_r}{\partial t} - \frac{k}{\phi\mu(\alpha + \beta)} \nabla^2 P_r = 0 \quad [4.28]$$

where α is solid-part compressibility, ρ_0 nominal fluid density, k medium permeability, other symbols as defined above. The grouping weighting the acceleration term $\partial^2 P_r / \partial t^2$ in Eq-4.28 has dimension [time] and so can be interpreted as a relaxation time for inertial pressure. For solid and fluid materials of interest here: $\rho_0 \sim 10^3 \text{ kg/m}^3$, $\mu \sim 10^{-3} \text{ Pa-s}$ (water), $\phi \sim .1$, $k \sim 10^{-15} \text{ m}^2$ [RIS-based estimates], giving a relaxation time of about 10^{-8} s to which corresponds a frequency of 100 MHz. In contrast, the factor weighting the Laplacian term in Eq-4.28 is ~ 0.4 , with $\alpha \sim .25 \times 10^{-10} \text{ Pa}^{-1}$ (rock) and $\beta \sim 5 \times 10^{-10} \text{ Pa}^{-1}$ (water). Therefore, given that permeability k embodies pore geometry, inertial effects again appear quite negligible at seismic frequencies, even for pores geometrically different from characteristic cracks.

Thus, the inertial term in Eq-4.28 can be dropped giving the (diffusive) governing equation for ring-fluid pressure

$$\frac{\partial P_r}{\partial t} - \frac{k}{\phi\mu(\alpha + \beta)} \nabla^2 P_r = 0 \quad [4.29]$$

In cylindrical coordinates Eq-4.29, with $P_r = p_r e^{i\omega t}$, becomes

$$\frac{\partial^2 p_r}{\partial r^2} + \frac{1}{r} \frac{\partial p_r}{\partial r} + \gamma^2 p_r = 0 \quad [4.30]$$

where parameter γ is defined by

$$\gamma = \sqrt{-i} \sqrt{\frac{\phi\mu(\alpha + \beta)}{k}} \cdot \omega \quad [4.31]$$

Eq-4.30 is an homogeneous Bessel equation of order zero, for which the general solution can be written

$$p_r(r, \omega) = c_3 J_0(\gamma r) + c_4 Y_0(\gamma r) \quad [4.32]$$

where c_3 and c_4 are two new constants of the problem. Unlike the passive ring, constant c_4 cannot be set to zero on grounds of Eq-4.19 since the gap-ring junction lies at $r = a$, a finite value. However, since the ring is spatially limited, a no-flow condition can be imposed at $r = R$, giving the boundary condition

$$\left. \frac{\partial p_r}{\partial r} \right|_{r=R} = 0 \quad [4.33]$$

Using Eq-4.33 to eliminate c_4 in Eq-4.32 leads to

$$p_r(r, \omega) = c_3 \left[J_0(\gamma r) - \frac{J_1(\gamma R)}{Y_1(\gamma R)} Y_0(\gamma r) \right] \quad [4.34]$$

where indices of Bessel functions refer to their order. Pressure continuity at $r = a$ (Eq-4.14) can be stated here

$$c_0 + c_1 J_0(\kappa a) = c_3 \left[J_0(\gamma a) - \frac{J_1(\gamma R)}{Y_1(\gamma R)} Y_0(\gamma a) \right] \quad [4.35]$$

where constant c_0 is still given by Eq-4.21 and so is known. Coupled arbitrary constants c_1 and c_3 can be determined using Eq-4.35 together with the flow-rate equality condition at $r = a$, provided $q_r(a, \omega)$ is made explicit in Eq-4.17. To this end, Darcy's empirical relation between the filtration velocity of a viscous fluid in a medium of permeability k and pressure-gradient $\partial p_r / \partial r$ across it can be used. In terms of volumetric flow-rate $q_r(r, \omega)$, Darcy's law can be written

$$q_r(r, \omega) = -A \left(\frac{k}{\mu} \right) \frac{\partial p_r}{\partial r} \quad [4.36]$$

where $p_r = p_r(r, \omega)$ as given by Eq-4.34, and A is the cross-area through which fluid flows in the direction of decreasing pressure. Noting that $A = 2\pi a h_0$ at the gap-ring junction, flow-rate continuity at $r = a$ can be formulated using Eq-4.17 and Eq-4.36, yielding

$$i\omega [\pi a^2 \zeta h_0 + \pi a^2 h_0 \beta p_g(a, \omega)] = -2\pi a h_0 \left(\frac{k}{\mu} \right) \frac{\partial p_r}{\partial r} \bigg|_{r=a} \quad [4.37]$$

Expressing $p_g(a, \omega)$ as load per gap-surface area via Eq-4.24, and using Eq-4.21 for c_0 , Eq-4.37 reads

$$-i\omega \frac{a c_0 \beta}{2} + \frac{i\omega \beta}{a} \int_0^a p_g(r, \omega) r dr = - \left(\frac{k}{\mu} \right) \frac{\partial p_r}{\partial r} \bigg|_{r=a} \quad [4.38]$$

where the first term on the left-hand side reflects gap-wall velocity, the second the associated compressibility effect. The right-hand term is filtration velocity in and out of the ring.

Solving for c_1 and c_3 in Eq-4.35 and Eq-4.38, I find for pressure-flow in the gap

$$p_g(r, \omega) = -\frac{\zeta}{\beta} \cdot \left[1 - \frac{J_0(\kappa r)}{J_0(\kappa a) - \Omega J_1(\kappa a)} \right] \quad [4.39]$$

where Ω is a frequency-dependent quantity embodying ring-material properties, given by

$$\Omega = \frac{i\mu\beta}{k} \cdot \Delta_R \frac{\omega}{\kappa\gamma} \quad [4.40]$$

with

$$\Delta_R = \frac{J_0(\gamma a) - \frac{J_1(\gamma R)}{Y_1(\gamma R)} Y_0(\gamma a)}{J_1(\gamma a) - \frac{J_1(\gamma R)}{Y_1(\gamma R)} Y_1(\gamma a)} \quad [4.41]$$

The complex pressure-field of Eq-4.39, for squeeze-flow in a cell with active ring, has real and imaginary parts which also can be related to dissipation as shown next.

4.3 Dissipative cell stiffnesses

The pressure equation for the cell with passive ring (Eq-4.25) and that for the cell with active ring (Eq-4.39) have no immediate physical interpretation. However, considering that viscous dissipation occurs as a result of the overall stress-response of the gap, both equations can be integrated over gap radius, leading to a definition for cell stiffness which is amenable to a clear physical interpretation. Thus, using Eq-4.24 to integrate both Eq-4.25 and Eq-4.39, and dividing the result by ζh_0 , the complex cell stiffness, in units of [Nt/m], is defined as

$$\frac{F}{\zeta h_0} = S_{P,A}(\omega) \equiv \text{Re}[S_{P,A}(\omega)] + i\text{Im}[S_{P,A}(\omega)] \quad [4.42]$$

to which correspond a phase lag between load F and fractional displacement ζh_0 given by

$$\theta_{P,A} = \text{ArcTan} \left[\frac{\text{Im}[S_{P,A}(\omega)]}{\text{Re}[S_{P,A}(\omega)]} \right] \quad [4.43]$$

Thus, dissipation caused by the retarding viscous-flow can be associated to the loss tangent: $\tan \theta_{P,A}$. Also, stiffness magnitude can be calculated from $\{[\text{Re}(S_{P,A}(\omega))]^2 + [\text{Im}(S_{P,A}(\omega))]^2\}^{1/2}$. $F/\zeta h_0$ in Eq-4.42 is a complex, frequency-dependent quantity whose components can be interpreted similarly to a mechanical impedance⁴.

Carrying-out the calculation for stiffness $S_{P,A}(\omega)$ yields: for the cell with passive ring,

⁴ Some authors (e. g. Murphy et al., 1986) call stiffness $F/\zeta h_0$ impedance, but this is wrong since mechanical impedance is force per velocity by definition. However, attributes of stiffness in the frequency domain are akin to those of impedance.

$$S_P(\omega) = \frac{\pi a^2}{h_0 \beta} \cdot \left[1 - \frac{2\Theta_P J_1(\kappa a)}{2\pi a^2 h_0 J_1(\kappa a) + \kappa a \Theta_P J_0(\kappa a)} \right] \quad [4.44]$$

; and for the cell with active ring,

$$S_A(\omega) = \frac{\pi a^2}{h_0 \beta} \cdot \left[1 - \frac{2\Theta_A J_1(\kappa a)}{a J_1(\kappa a) + \kappa a \Theta_A J_0(\kappa a)} \right] \quad [4.45]$$

where

$$\Theta_A = \frac{i}{\beta \mu \Delta_R} \cdot \frac{k\gamma}{\omega} \quad [4.46]$$

It is seen from comparing Eq-4.44 to Eq-4.45 that, unlike the passive ring constant factor Θ_P , the active ring factor Θ_A is both complex and frequency-dependent.

4.3.1 Stiffness results

I calculated crack stiffness for both cell models by implementing Eq-4.44 and Eq-4.45 in Wolfram's (1988) *Mathematica*TM software, installed on a NeXTTM computer. This symbolic, mathematical software allows easy separation of real and imaginary parts of the complex stiffnesses and so computation of the phase lags. For illustrating the physics of dissipation by squeeze-flow in the seismic band I used parameters appropriate for characteristic cracks in a three-layer crustal model, as described in Table-4.1. Only the bandwidth 0.1 to 100 Hz was considered as it is that through which significant manifestations of rock-water interaction are sought. The lowermost crustal layer [20 — 35 km] is not modelled since temperature there is such that the rock matrix likely departs from brittle behaviour.

nesses shown are for the mid-layer parameters [4 — 10km] of Table-4.1, the other two sets giving identical qualitative features. Components of the calculated stiffnesses were referenced to their value at the origin, setting zero frequency at 10^{-6} Hz, then normalised to the real part of $S_{P,A}(\omega)$ at infinite frequency, taken at 10 MHz. Crack stiffness $S_{P,A}(\omega)$ represents the ability of the cell to oppose cyclic loading of the gap-walls as well as the mobility of its fluid content during the cycle. The real part of $S_{P,A}(\omega)$ determines the amount of compression/dilation in the squeezing fluid; the imaginary part, when significant, determines viscous flow behind loading and so energy dissipation. As for any linear absorbing medium, real and imaginary components obey the Kramers-Krönig dispersion relations, giving the latter (absorption) as essentially the slope of the former (dissipation) [e. g. Brennan and Smylie, 1981].

The principal feature of both stiffnesses is the significant dissipation and dispersion well within the crustal seismic band. At high frequencies, loading and fluid compression/dilation are in phase; therefore, fluid elasticity is the only compliant component of cell-stiffness. Indeed, normalisation factors approach the value of the common constant multiplying the bracketed terms in Eq-4.44 and Eq-4.45 (the limit as $\omega \rightarrow \infty$ of the respective frequency-dependent part in these equations tends to zero). At low frequency, compression/dilation of the fluid lattice is practically infinitesimal and so out-of-phase with the load. At intermediate frequencies real and imaginary parts share the stiffness amplitude response; in particular, dissipation is maximal at a well-defined characteristic frequency.

The two stiffnesses differ in magnitude and dispersion character. As expected, dissipation is greater for the active-ring cell in which the flow caused by the pressure-gradient imposed across the ring contributes more losses. Dispersion for the active-ring cell appears "anomalous" about the characteristic frequency, since it is larger than

Table 4. 1: Cell model parameters for characteristic crustal cracks

Depth-range [km]	0 — 4	4 — 10	10 — 20
Gap radius a [m]	$10. \times 10^{-3}$	1.0×10^{-3}	1.0×10^{-3}
Ring radius b (passive) [m]	5.0×10^{-3}	0.5×10^{-3}	0.5×10^{-3}
Ring radius R (active) [m]	$20. \times 10^{-3}$	2.0×10^{-3}	2.0×10^{-3}
Gap thickness h_0 [m]	$20. \times 10^{-9}$	2.0×10^{-9}	1.0×10^{-9}
Fluid compressibility β [Pa^{-1}]	4.6×10^{-10}	4.6×10^{-10}	4.6×10^{-10}
Solid compressibility α [Pa^{-1}]	$.44 \times 10^{-10}$	$.25 \times 10^{-10}$	$.20 \times 10^{-10}$
Ring porosity ϕ [%]	$20. \times 10^{-2}$	$15. \times 10^{-2}$	$10. \times 10^{-2}$
Ring permeability k [m^2]	100×10^{-15}	$10. \times 10^{-15}$	1.0×10^{-15}
Fluid viscosity μ [$\text{Pa}\cdot\text{s}$]	1.0×10^{-3}	0.6×10^{-3}	0.3×10^{-3}

N.B. ϕ and k values estimated from Talwani & Acree 's (1985) reservoir-induced seismicity study.

Figure-4.8 shows the results for the cell with passive ring, figure-4.9 for the cell with active ring. Both stiff-

the stiffness at infinite frequency. This feature, commonly observed in physical optics and analogous experiments,

suggests a form of resonance perhaps generated by flow feedback from the ring itself. This resonance, however, is eliminated when solid-solid contacts between cell walls are included in the total response (cf. chapter-5).

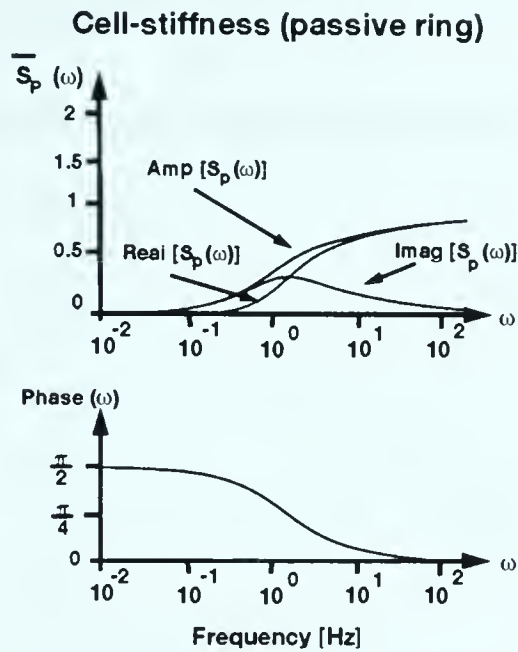


Figure-4.8 Components of complex stiffness for cell with passive ring $\bar{S}_p(\omega)$. Bar denotes normalisation to real part of $S_p(\omega)$ at infinite frequencies, after referencing each component to corresponding value at axis origin. Phase angle represents lag of stiffness response relative to instantaneous compression. Viscous flow is maximal a phase angle $\pi/4$. Bandwidth shown covers that of crustal seismics.

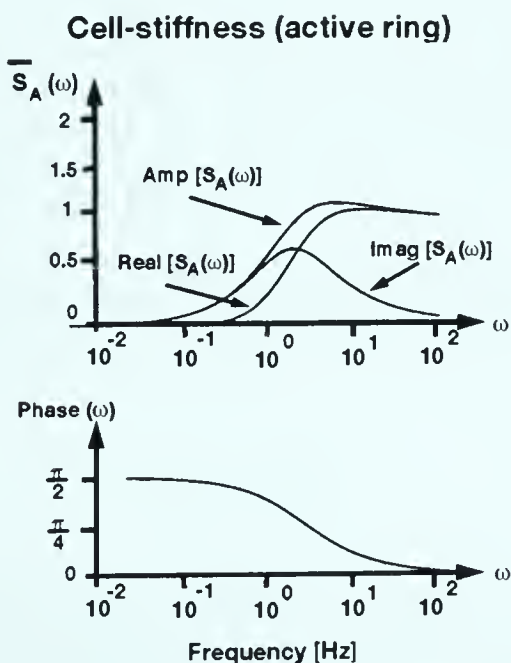


Figure-4.9 Components of complex stiffness for cell with active ring $\bar{S}_A(\omega)$. Normalisation and phase description as in caption of fig-4.8

4.3.2 Parameter sensitivity

Figure-4.10 shows the quadrature (imaginary) component of both types of stiffness for the three sets of parameters in Table-4.1. This result can be viewed as a depth-distribution of dissipation in the crust, assuming that the

density of characteristic cracks is roughly the same in each layer. Figure-4.10 also shows some sensitivity of cell stiffnesses to model parameters. It is seen that, despite large parameter changes, peak dissipations remain significant and well within the crustal seismic bandwidth, even down to 20 km. Dissipation for the active-ring cell is everywhere larger than for the passive-ring, with a characteristic frequency more sensitive to parameter variations.

In order to examine more closely which parameters influence cell-stiffnesses most, I calculated quadratures for the mid-layer values of both cell models by varying, in turn: crack aspect-ratio ϵ , gap length a , gap thickness h_0 , and fluid viscosity μ . I also varied ring permeability k , porosity ϕ , and radius R for the active-ring model. Parameters a , R , and b were varied accordingly to the geometrical constraint $R > a$ for the active ring, and $a > b \sim a/2$ for the passive ring. Application of the latter constraint effectively changes the fluid-storing capacity of the passive ring and so reflects porosity variation as well. Viscosity μ was assumed a function of temperature T , with μ values taken from Kestin et al.'s (1981) tabulation for brines⁵ and T values estimated from Shankland and Ander's (1983) crustal depth-profiles.

Figure-4.11 shows dissipation amplitude in units of decibel and characteristic-frequency shift in relation to crack aspect-ratio, for both cell models. It is seen that dissipation scales downward with increasing aspect-ratio and that peak values shift to higher frequencies in the same respect. This frequency shift can be

Depth-distribution of cell-stiffness quadratures

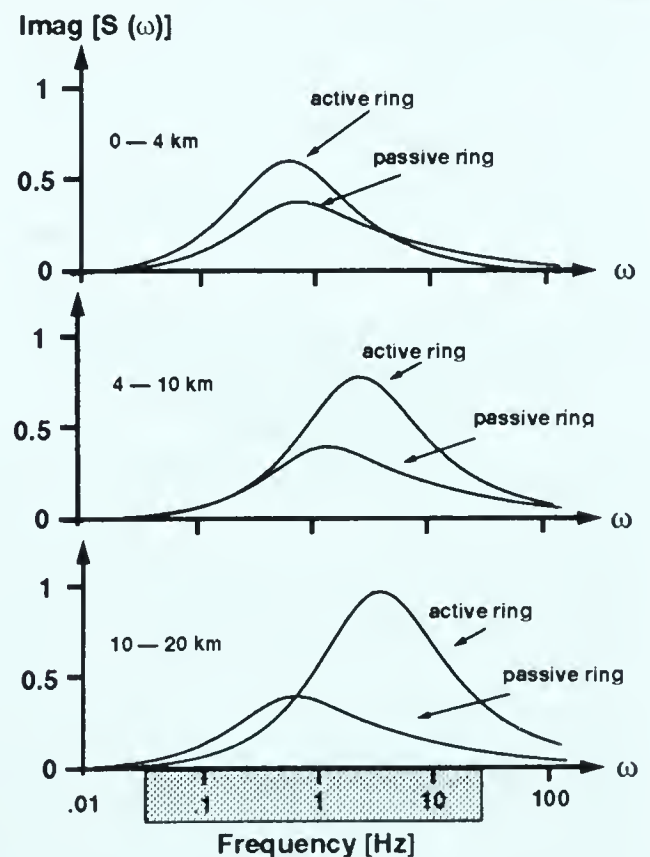


Figure-4.10 Cell stiffness quadratures of active and pas-

⁵ Kestin et al.'s tabulation is based on an Arrhenius equation giving viscosity as an exponentially decreasing function of inverse temperature.

sive ring models, for each layer parameterised in Table-4.1. Quadratures are referenced to their values at axis origin, then normalised to corresponding real part of stiffness at infinite frequency. Shaded bandwidth is that covered by measurements of seismic dissipation at the surface of the Earth.

Cell stiffness quadrature and crack aspect-ratio

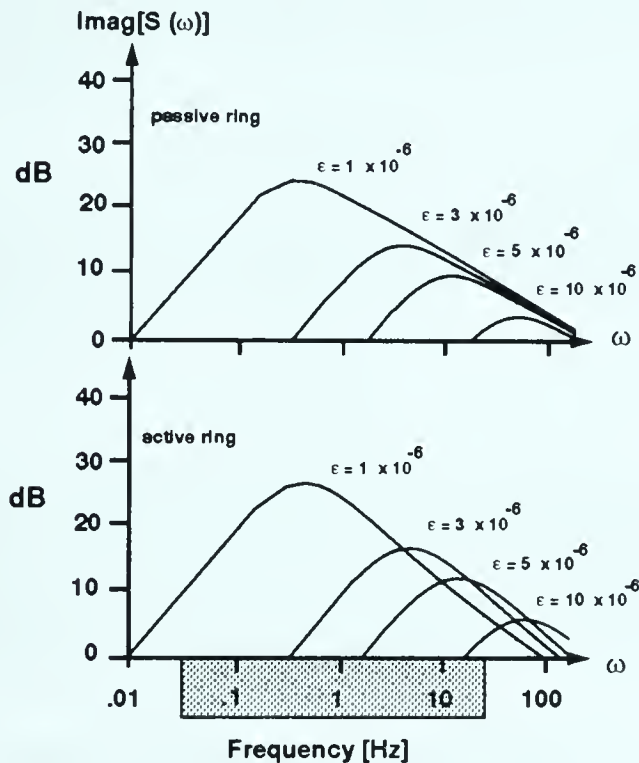


Figure-4.11 Stiffness quadratures of passive and active ring models for various gap aspect-ratios ϵ . Unaltered parameters are those for the mid-layer of Table-4.1. Quadrature amplitudes in decibels, relative to quadrature value at axis origin when $\epsilon = 10^{-6}$. Shaded bandwidth is that spanned by measurements of seismic attenuation at the surface of the Earth.

explained on the basis of a crude estimate for squirt-flow characteristic frequency f_c , calculated by O'Connell and Budiansky (1977)⁶. For the problem here, f_c can be written

$$f_c = \frac{1}{\beta\mu} \cdot \left(\frac{h_0}{a}\right)^3 \quad [4.47]$$

Clearly, as aspect-ratio h_0/a increases so does f_c . As to the severe decrease in dissipation magnitude, it can be explained using Eq-4.10 which indicates that the term driving gap-fluid pressure is inversely proportional to the cube of gap thickness h_0 . Thus, larger aspect-ratio cracks can be expected to produce little dissipation in the crustal seismic band.

⁶. This f_c is given by equation-A30 of their paper, obtained by assuming that the rate of fluid-flow in and out of two moving parallel plates is proportional to the fluid pressure induced by normal stresses acting on the plate surfaces.

The similarities between active-ring and passive-ring cell stiffnesses exhibited in figure-4.11 appeared in all the parameter variations performed. For brevity, only results for the active-ring cell are presented hereon. Figure-4.12 shows stiffness quadrature variation resulting from changing gap-length a and gap-thickness h_0 , in turn. It is seen that as the gap-length increases dissipation augments and shifts towards lower frequencies, in accordance with Eq-4.47 and with the fact that a longer thin-film generates correspondingly more viscous losses. Similarly, as gap-thickness decreases, shearing of fluid layers aggravates and dissipation peaks at lower frequencies.

Figure-4.13 shows quadrature-phase cell-stiffness when fluid viscosity and ring porosity are altered. Again in accordance with Eq-4.47, maximal dissipation shifts to higher frequencies as viscosity decreases or, equivalently, as temperature increases. Dissipation magnitude is not noticeably affected within the range of viscosity variation expected for crustal brines ($\sim$ a factor 10); however, the

Stiffness quadrature and gap dimension (active ring)

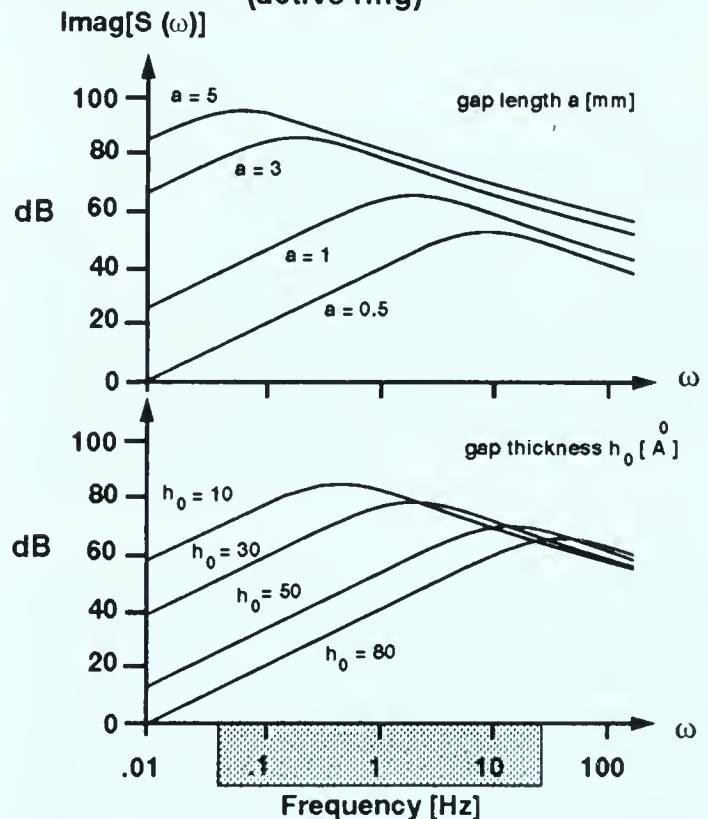


Figure-4.12 Stiffness quadrature of active-ring cell for various gap-lengths and gap-thicknesses, other parameters (mid-column Table-4.1) kept constant. Decibel scale is established as described in fig-4.11's caption. Respective reference values given by parametric curve starting at the origin. Shaded bandwidth = crustal seismic band.

corresponding frequency shift is quite significant, reflecting sensitivity of the squeeze-flow model to fluid temperature. Ring porosity variation also has a large effect on dissipation, giving larger magnitudes for larger porosities with corresponding shifts towards lower frequencies. This is expected since a larger porosity enhances fluid exchanges between gap and ring; therefore, more viscous losses. The frequency shift can be explained by Eq-4.47, with gap-length as being effectively larger due to fluid penetration in the ring.

Figure-4.14 shows cell-stiffness quadratures when ring radius R and ring permeability k are altered. Here, quadrature variations are slight relative to all other parameter changes, thereby showing little sensitivity to ring material properties other than porosity. This somewhat deceiving result, particularly for ring permeability, is not surprising, however, when considering the relative size of diffusion lengths associated with the diffusivity coefficients implicit in the gap-fluid and ring-fluid governing equations. The diffusivity coefficient, D_r in units of $[m^2/s]$, for excess fluid pressure in the ring is given by the grouping which weights the Laplacian term in Eq-4.29, namely,

$$D_r = \frac{k}{\phi\mu(\alpha + \beta)} \quad [4.48]$$

The coefficient D_g corresponding to excess fluid pressure in the gap is obtained by rearranging the time-domain form of Eq-4.10, which leads to

$$D_g = \frac{h_0^2}{12\mu\beta} \quad [4.49]$$

Since diffusivity, by definition, embodies the length-squared through which excess pressure relaxes during time t , the square-root of D_r or D_g can be interpreted as a diffusion distance which varies with the square-root of time (as in all random processes). Here, the question is whether excess pressure, which diffuses

porosity variations inferred from RIS observations.- Shaded bandwidth = crustal seismic band.

Stiffness quadrature for active-ring cell (ring radius and permeability)

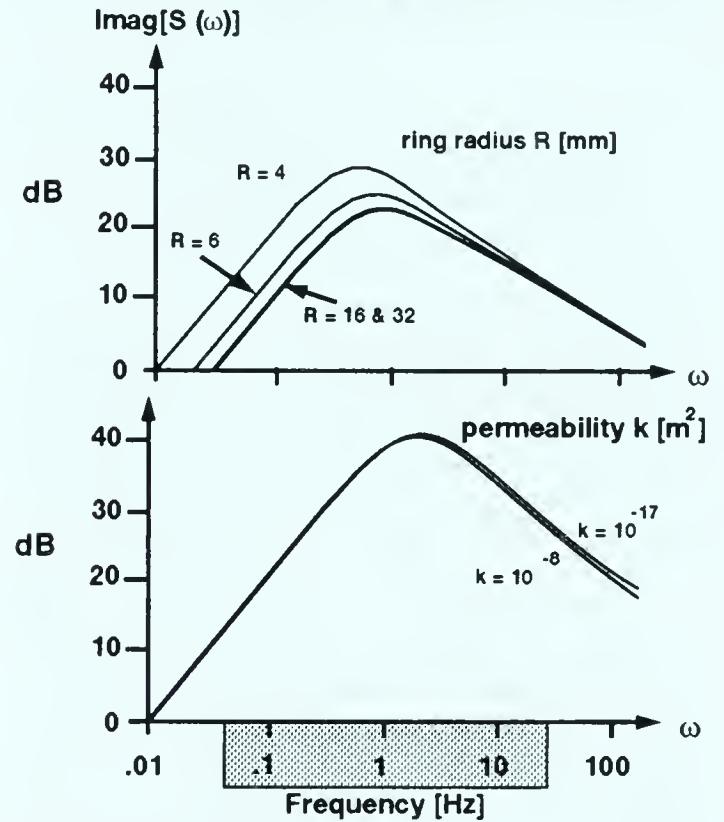


Figure-4.14 Stiffness quadrature of active-ring cell for various ring radii R and permeabilities k , other parameters (mid-column Table-4.1) kept constant. Decibel scale established as in fig-4.11's caption, with respective references given by curve starting at the origin. Shaded bandwidth = crustal seismic band.

Stiffness quadrature for active-ring cell (fluid viscosity and ring porosity)

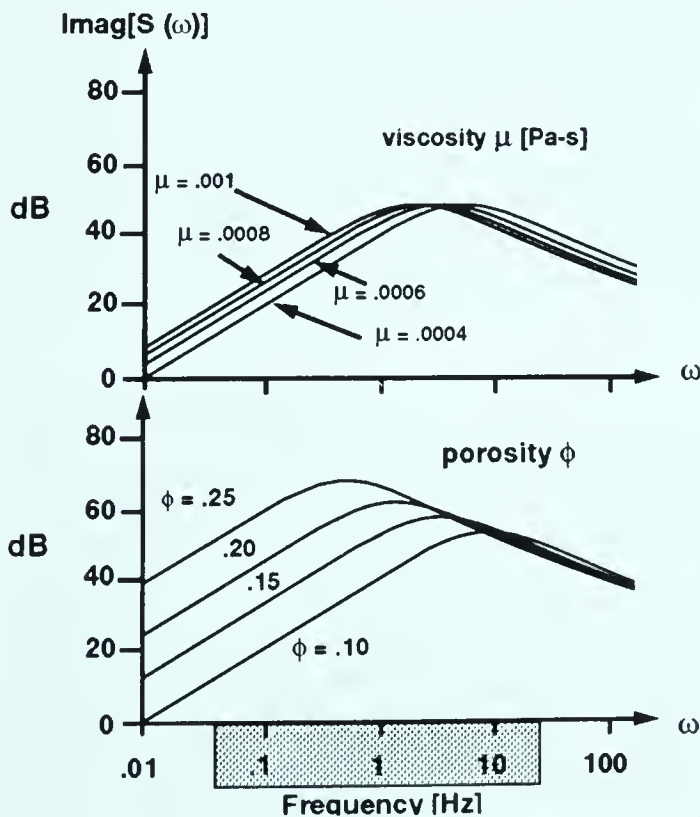


Figure-4.13 Stiffness quadrature of active-ring cell for various fluid viscosities and ring porosities, other parameters (mid-column Table-4.1) kept constant. Decibel scale established as in fig-4.11's caption, with respective references given by curve starting at the origin. Variation of viscosity is for saline water at crustal temperatures. Ring

in the form of viscous flow, is efficient during fluid squeezing, given the space of the gap and ring. Taking $\sqrt{D_r} = l_r/\sqrt{t}$ and $\sqrt{D_g} = l_g/\sqrt{t}$, and using the parameters of Table-4.1, the ratio of ring diffusion-length l_r to gap diffusion-length l_g typically gives $l_r/l_g \sim 500$, with $l_r \sim 500$ mm and $l_g \sim$ one mm. Thus, the difference in diffusion-lengths is very large; moreover, $l_r \gg R$ - a, the length of the ring, and $l_g \sim a$, the length of the gap. It is clear then that viscous flow is much more efficient in the gap, a result which can be considered as *a posteriori* support to the view that the central element of squeeze-flow is the gap, with the ring acting as moderator. It is also apparent from this result that gap-fluid does not diffuse much into the ring, thereby indicating that for seismic oscillatory squeezing the fluid does not have time to fully "sense" ring properties such as permeability. This lack of sensitivity is further illustrated, mathematically, by the weak frequency-dependence of factor Θ_A in the active-ring cell stiffness (Eq-4.45) for relevant parameters, although its corresponding modulus always exceeds the constant factor Θ_P in the passive-ring cell stiffness (Eq-4.44).

In summary, both cell models are most sensitive to gap parameters ϵ , a , h_0 , to fluid viscosity μ (and temperature), and to ring parameter ϕ . This sensitivity is characterised

by a frequency-shift of maximal dissipation accompanied by a diminution of dissipation magnitude, as illustrated in figures-4.10, 4.11, 4.12, and 4.13. The frequency-shift differs according to the type of boundary conditions applied at the gap periphery, being more severe for the active-ring cell. In all cases, however, dissipation is larger for the active-ring cell, and most peak dissipation values remain significant within the crustal seismic band, down to depths ~ 20 km. Sensitivity to ring parameter properties other than porosity is weak, but the overall effect on gap-fluid flow remains larger for the active-ring cell than for the passive-ring.

It may be argued at this point that, given the variability of fracture-interspace geometry, the sum total of all possible dissipation values would yield a smeared frequency dependence of dissipation in the bandwidth of interest, as suggested by superposing the various stiffness quadratures shown in the above-mentioned figures. This would be true for a fractured medium in which the crack density of each calculated cell is equal. However, this argument is weak because the main feature of natural fractures is the likely existence of one "typical" crack which, by virtue of its abundance, has the largest crack density and so can logically be expected to dominate the squeeze-flow response. The combination of Scholz' (1988) fracture-topography constraints and the physics of squeeze-flow developed here indicate that such a characteristic crack corresponds to that with the lowest thickness, for which dissipation is maximal, significant, and with peak values well-within the crustal seismic band.

With respect to the different frequency-shift responses of the active and passive ring cells, it may be argued again that superposition of their respective stiffness quadrature could smear the frequency-dependence of the overall dissipation; however, the active-ring model, being more consistent with the concept of localised heterogeneous stress, can be considered more representative of the interspace of natural fractures. Besides, active-ring cell dissipation, being always larger than that for the passive-ring, would logically dominate the frequency-dependent dissipation response. Therefore, it is reasonable to deduce that the squeeze-flow mechanism developed here has a robust frequency selectivity, within the crustal seismic band.

4.4 Conclusion

I have shown that, when crustal formations are regarded as a packing of corrugated, impermeable blocks, oscillatory squeeze-flow of a compressible viscous fluid in a crack which characterises block-contact interspace generates significant, frequency-dependent dissipation of mechanical energy. This viewpoint has led to a new result in that dissipation peaks well within the crustal seismic bandwidth, even down to depths ~ 20 km. This contrasts with previous squirt-flow results which produce peak dissipation in the kHz range and so of no applicability to seismic dissipation in the crust.

My result is founded on the microgeometry representation of a characteristic crack, constrained by fracture-topography measurements and by field-based inferences of related hydraulic properties. Of the two models considered for the periphery of such a characteristic crack, a

passive and an active bounding ring, the latter appears more appropriate for the squeeze-flow response of natural fractures. The mechanism of dissipation by squeeze-flow developed here exhibits a robust frequency selectivity for the range of possible parameter values and so provides a basis upon which an explanation for the apparent attenuation observed at the surface of the crust can be built.

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Chapter-5: Seismic attenuation models for an hydrated crust¹

Interest in the problem of characterising attenuation in the Earth from surface measurements has recently been stimulated in crustal seismology, in a large part due to the advent of modern broadband seismometers which have led to high-quality data up to tens of Hz for several tectonic areas. At such high frequencies many structural details of the crust become manifest in the seismic response, a phenomenon which has generated considerable activity in the field of seismic scattering. It is now established that attenuation observed at the surface of the Earth in the form of the inverse of the total quality factor — $Q_t^{-1}(\omega)$ — ubiquitously exhibits strong frequency (ω) dependence in the band 1 — 25 Hz, with a continual decrease above one Hz (Sato, 1990). Comparison of high-frequency Q_t^{-1} observations with collocated data from lower frequency surface-waves trapped in the crustal waveguide suggests the presence of a characteristic frequency round one Hz, about which Q_t^{-1} asymmetrically diminishes in the band 0.08 — 25 Hz (cf. chapter-2). What is not established, however, is the source of such an attenuation response. Although elastic scattering by randomly distributed material inhomogeneities is a prime candidate (i.e. scattering Q_{sc}^{-1}), intrinsic attenuation (i.e. Q^{-1}) may also contribute markedly to the response, especially if the state of hydration of the crust is such as to favour energy losses by viscous squeeze-flow of interstitial fluids as I have modelled in chapter-4.

In parallel with this development new geological and geophysical constraints on the heterogeneous and hydraulic nature of crustal materials has become available, the relevancy of which largely remains to be assessed in the context of seismic attenuation. Current explanations for the crustal-seismic Q_t^{-1} signature include: 1) wave scattering by large-scale randomly distributed impedances (Sato, 1990); 2) depth-distribution of intrinsic attenuation (Cong & Mitchell, 1988; Mitchell, 1991); and 3) combinations of both intrinsic and scattering mechanisms (Toksöz, Mandal & Dainty, 1990). All three explanations may be related to fluid-rock interactions of various kinds; for instance, scattering from randomly distributed dry/saturated zones, energy loss from viscous flow of crustal fluids within a permeable rock-frame, or destructive interference from dry and saturated thin layers.

Much of this interpretation work has been conducted from a purely seismological standpoint, with the main

effort directed at inferring the general character of *in situ* attenuation from surface Q_t^{-1} data. Whilst this is a laudable and very demanding step — still far from being completed — it eventually provides only gross information about the physics of attenuation. Ideally, attenuation maps should be obtained from inversion techniques applied over the full observational bandwidth, but these are poorly developed for high-frequency Q_t^{-1} data and can hardly resolve the ambiguity associated with a medium such as the crust which both scatters and dissipates wave energy. Thus, at present, hope for elucidating *in situ* attenuation lies in forward techniques. Within these limits, further progress requires explicit models of possible spatial distributions of attenuation, covering the entire observational bandwidth and embodying the most likely dominating mechanisms of seismic energy losses — dissipation and scattering.

I submit, here, a first attempt at modelling the depth-distribution of attenuation in an hydrated crust, whilst respecting available constraints on the heterogeneous and hydraulic character of the continental crust. Emphasis is intentionally put on intrinsic attenuation (i.e. dissipation) by fluid/rock interaction mechanisms — squeeze-flow in particular, as developed in chapter-4. My approach is motivated by the fact that this is the least investigated aspect of crustal-seismic attenuation, yet the one most constrained by ancillary results. The main purpose is to examine the magnitude and frequency-dependence of attenuation as thermodynamic conditions vary with depth. Results are judged meaningful inasmuch as the modelled attenuation at a given depth exhibits a bell-shaped function of frequency of significant magnitude in the crustal-seismic band.

The proposed attenuation maps are schematic in that they do not characterise a specific tectonic area, but rather illustrate the likely nature and character of attenuation in a representation of the crust which accounts for the natural hydraulic segregation imposed by different rheological regimes as depth increases. This reference hydrated crustal model is structured accordingly to a brittle upper part, a transitional mid-layer, and a ductile bottom, each portion being hydraulically isolated from the other by impermeable boundaries. Thus, the applicability of the attenuation models to a specific area for which broadband Q_t^{-1} data are available can be examined using either single-layer results or combinations therefrom.

As strict guidelines for this quantitative modelling, the following rheological and material attributes are imposed *ab initio*: 1) a fractured upper crust whose impermeable rock-fragments are in rough contacts, with water between them; 2) a transitional zone whose upper part is sub-horizontally fractured, with thin saturated pore-networks nearing lithostatic pressure; and 3) a laterally heterogeneous lower crust whose acoustic impedances are associated with overpressured, brine-saturated lamellae of the type inferred from correlations between low-resistivity data and seismic-reflector images (e. g. Merzer & Klemperer, 1992). This endeavour may appear ambitious; however, by drawing from recent theoretical and experimental results it is possible to obtain constrained estimates for *in situ* seismic attenuation, at least for the continental crust.

In section-5.1, I elaborate on the rationale behind the reference crustal structure used and provide visualisations

¹. An earlier version of this paper was presented at the Spring meeting of the American Geophysical Union in Montréal, Québec on May 13th, 1992 (paper # S31C-5, Rouleau, P., Seismic dissipation by thin-film squirt-flow: two models for crustal rocks *in situ*).

for the various states of hydration which follow from this structure. The sharp thermodynamic differentiation at the $400 \pm 50^\circ\text{C}$ isotherm level, corresponding approximately to the critical point of crustal brines, is recognised which necessitates treating the lower crust differently from other parts. In section-5.2, I apply (and correct) an effective-medium technique proposed by Winkler (1983) and modified by Palciauskas (1992) for wave-propagation in small-scale granular rocks to crustal-scale brittle formations. In this technique, the medium is treated as a linear viscoelastic body which allows estimating dissipation at the macroscopic scale from complex moduli of stress-strain relations for rough rock-contacts. In section-5.3, I present dissipation/dispersion models for liquid-saturated brittle layers, based on estimates of the macroscopic parameters needed to compute the complex moduli and of the microscopic parameters associated with the squeeze-flow mechanism described in chapter-4. Inferences relying on the poorly-constrained "crack-density" parameter (e. g. O'Connell & Budiansky, 1977) are deliberately avoided, in favour of permeable-media models for which there is a reliable experimental basis. Squeeze-flow results are compared with Biot's bulk-flow mechanism (Biot, 1956). I also show that significant dissipation by localised squeeze-flow is unlikely for the lower crust, whereas scattering attenuation may occur.

5.1 Rationale for an hydrated crustal model

As discussed in chapter-2, deep-drilling results, reservoir-induced seismicity occurrences, extensive-dilatancy anisotropy observations, collocated seismic reflection and electrical resistivity profiles, and geochemical studies of prograde metamorphism provide good evidence that the Earth's crystalline crust is pervasively hydrated by H_2O -rich fluids, preferentially residing within the interspace of natural fractures. Combining these results, together with constraints from structural geology and rock rheology, suggests the hydrated crustal structure representation schematised in figure-5.1. This composite crustal model is divided in three layers, each characterised by nominal rock composition and rheological configuration. Viewed at the crustal scale (left, fig-5.1) each of the three layers appears as a macroscopically homogeneous medium, known to transmit seismic waves nearly elastically. At a finer scale (right), important structural heterogeneities can be envisioned, as shown. I briefly describe here the basis for this characterisation of an hydrated crust and its implications for modelling attenuation.

5.1.1 brittle upper crust

It is often remarked in seismic-attenuation studies that the upper crust is fractured over many scales which calls for some recognition of an intrinsically fragmented structure, arguably throughout the seismogenic zone (depth < 20 km). Such structure can be idealised in two dimensions as an honeycomb pattern composed of regular hexagons (figure-5.1). The gross mechanical behaviour of such a structure helps in distinguishing which properties most affect wave-propagation and attenuation. Turcotte

(1986) used a trio of hexagons as an elementary unit for constructing a multiple-order fractal network in order to model strike-slip faulting throughout the San Andreas fault system. Using an earthquake frequency-magnitude relation to estimate the system's fractal dimension, he showed that roughly half the creep expected from the relative motion of the Pacific and North American plates is accommodated by primary (first-order) large faults of size ~ 10 km, the other half being taken up by higher-order smaller faults, a great many less than 100 m. He also showed that faulting imparts differential rotation of each hexagon, thereby generating small void-spaces at displaced corners which eventually fill with exsolved minerals of large compliance. His work suggests that contiguous sides of solid-fragments of a few to several tens of metres play a major role in the mechanical response of fractured crust, whereas minerals filling the space near displaced corners play a minor one.

This dimension of tens of metres or so, inferred from manifestations of higher-order faults, also comes from field studies of a more geological nature, albeit limited in depth penetration. Segall & Pollard (1983) reported length measurements of tensile joints for an outcrop of a fractured granite formation in the Sierra Nevada and found distributed dimensions, roughly peaking at $\sim 5 - 8$ m, but rarely exceeding 100 m. This dimensional range also appears in the vertical direction from deep-drilling logs in crystalline environments, at least down to a few km (e. g. Stone et al., 1989). Field-constrained correlations between fracture separation and bed thickness also suggest a block structure, with block-size of the order of a few m (Price & Cosgrove, 1990). Whether this field-constrained dimensional range is in some sense "characteristic" of the whole upper-crust is difficult to say; however, that fractured rock formations several km-thick are directly observed down to at least 12 km (Kozlovsky, 1987) certainly is a good indicator that fragmented rock and accompanying void space are not mere surficial features. It is also difficult to determine which portions of fracture interspace associated with rock-fragment boundaries may be dry, saturated or mineral-filled. However, in light of Turcotte's (1986) modelling results, it can be assumed that preferential *loci* for earthquakes generated by excess pore-fluid pressure along fault-walls coincide with higher-order sets of faults, which, by virtue of the fractal nature of the whole fault system, are the most numerous. Since these are of the order of tens of

Reference Structure for an Hydrated Crust

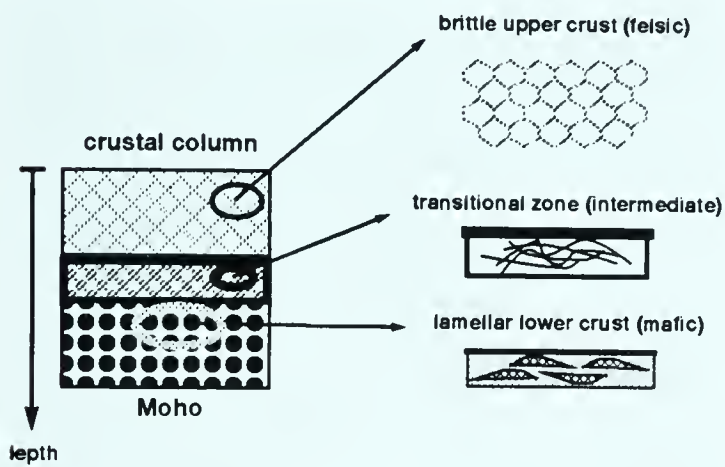


Figure-5.1 Reference structure for a layered and hydrated crust. Left: schematic crustal column, with each layer considered as a continuum at the macroscopic scale. Right: structural elements at a finer scale. Upper crust is block-structured, with fluids saturating block boundaries; fluid pressure may vary from hydrostatic to lithostatic. Top of transitional zone appears as a pattern of intersecting sub-horizontal fluid-filled fractures, within which fluids occur near lithostatic pressure. Lower crust is pervaded by a distribution of overpressured porous lamellae, in textural equilibrium with its ductile environment. Nominal rock composition in parentheses.

metres, this dimension will here be assumed characteristic of the brittle part of the fractured crust, as idealised in fig-5.1. This range of rock-fragment sizes is somewhat convenient from the viewpoint of seismic wave propagation in that structural inhomogeneities are sufficiently smaller than relevant wavelengths and so presumably generate little scattering (for a shear-wave velocity of 3.5 km/s at 25 Hz, the minimal wavelength is 140 m).

The hexagonal pattern shown in fig-5.1, by virtue of its geometrical regularity, implies strong anisotropy (Gas-smann, 1951), likely much stronger than thought to be observed in the crust (Crampin, 1987). Significant anisotropy is also at odds with Q_t^{-1} observations which generally do not show any significant azimuthal variations and so justify the simpler isotropic representation. Moreover, the two-dimensional honeycomb pattern in fig-5.1 is unrealistic since, over the evolution of the crust, it is likely that many tectonic events rather produce a three-dimensional packing of fragments of various dimensions. Wave propagation in such intricate structure is not amenable to a deterministic description; however, by assuming that fractured crust is structured as a random packing of like-fragments, isotropy follows and wave attributes can be characterised for geometrically simple fragment shapes.

The simplest shape is that of a sphere which can be envisioned as inscribing the periphery of each fragment of the structure (figure-5.2), thereby resulting in a random sphere-pack. Wave velocities in such structure have been formulated by Winkler (1983) and intrinsically depend on the mechanical stiffness of sphere contacts (Digby, 1981). This formulation is of particular interest here since it was

applied to model squirt-flow dissipation in granular rock-samples (Murphy et al., 1986) and since squeeze-flow dissipation is described in micromechanical-stiffness terms in chapter-4; however, its applicability to crustal-scale formations remains to be demonstrated. Although it may seem fanciful to regard brittle crust as a sphere-pack, it is not more so than treating it as a stack of perfectly elastic homogeneous layers as commonly assumed in conventional seismology.

The above considerations entail the scale hierarchy depicted in figure-5.3 in relation to seismic dissipation by squeeze-flow and suggests an operational framework for estimating Q^{-1} therefrom. Dimensions range from several km for crustal-layer thickness, through a few mm for asperity spacing in fractal fractures (Scholz, 1988), to 10^{-8} — 10^{-9} m for crack-thicknesses conducive to severe viscous shearing by squeeze-flow of thin fluid-films (chapter-4). Hence, the two-dimensional honeycomb pattern of fig-5.1 is abandoned in favour of a more realistic three-dimensional random pack of rock-fragments, with characteristic sizes of the order of tens of m.

Crustal block and inscribed sphere concept

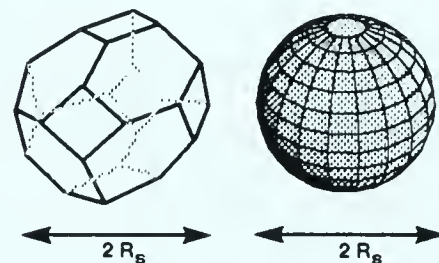


Figure-5.2 Assumed shape of characteristic crustal rock-fragment (left) and inscribing sphere (right). $2R_s$ denotes the associated characteristic dimension of the order of tens of m.

Thus, the operational framework within which the microscopic squeeze-flow mechanism can be scaled to the macroscopic Q^{-1} response of brittle crustal layers consists of assuming that: 1) the random pack of spherical fragments behaves as an isotropic homogeneous medium at the crustal-layer scale; 2) the packing is composed of perfectly elastic solid-fragments of fixed size and its nominal porosity is filled by highly compliant elastic solid-materials; wave attributes predominantly depend on fragment-fragment contacts, which are rough; 3) the packing porosity is reduced to that associated with rough-contact interspace, i.e. to fracture-porosity; and 4) the microstructure of fracture interspace is the site of seismic energy dissipation by squeeze-flow. It is emphasised that fracture-roughness is a key element, as it ensures that void-space does exist at great depth in the brittle crust, whether the ambient stress regime is dilatational or compressive (Gold & Soter, 1985).

5.1.2 Transitional zone and lower crust

The bottom layers of the hydrated-crust representation in fig-5.1 differ from the felsic upper-crust in many ways, especially with respect to the $400 \pm 50^\circ\text{C}$ isotherm with which the brittle/ductile boundary is often correlated. In particular, low electrical resistivity is ubiquitously observed in Phanerozoic areas at depths corresponding to this isotherm (e. g. see Marquis & Hyndman, 1992). Although this boundary marks an apparently sharp transition between two fundamentally different rheological regimes — brittle for the upper-crust, ductile for the lower crust — it seems an over-simplification to regard it as infinitely thin. A midlayer, sometimes associated with the "Conrad discontinuity", is often clearly identified from crustal-seismic surveys (e. g. Nur & Walder, 1990). Such layer is appropriately termed "transitional zone" whose rheology is likely intermediate between brittle and ductile.

For the hydrated lower crust, however, the demarcation of crustal properties, beyond the predominantly mafic composition, is clear: ambient temperatures are sufficiently high to impart ductility to rock materials; therefore, brittle-type fractures

domly oriented; passage of a shearing seismic-wave generates compressive stresses differentially localising at void walls which induce viscous flow confined to individual cells.

cannot be sustained over geologically long periods. Thermodynamic constraints also impose that pore-spaces assume a configuration of minimal surface energy, resulting in equilibrated textures characterised by smooth-surface rock-grains. Gross constraints on the hydrated state of the lower crust primarily come from correlating low electrical resistivity observations with collocated seismic reflectors imaged by continuous profiling. The interpretation of such correlation is non-unique; however, a coherent explanation is found by attributing low-resistivity to contiguous networks of grain-wetting saline-fluids trapped at depth, in either segregated homogeneous layers (Marquis & Hyndman, 1992) or in disconnected porous lamellae of seismic-reflector dimensions residing in a high-resistivity mafic background (Merzer & Klemperer, 1992; also see bottom layer of fig-5.1). Although this explanation requires long-time maintenance of pressurised free-fluid within impermeable boundaries of finite strength, which leads to gravitational-instability and metamorphism problems, there are realistic geodynamical processes conducive to its occurrence (Marquis & Hyndman, 1992).

Of interest here are the dimensional constraints on porous lamellae and intergrain porosity. Merzer & Klemperer (1992) pointed out that P-wave continuous profiles image reflectors averaging a few km in length and a few hundred meters in height, thereby forming a roughly 10 km-thick lower crust pervaded by unconnected overlapping lamellae of 10 — 15% impedance contrasts. However, collocated S-wave profiles of similar frequency coverage offer a different picture: impedance contrasts are much less (1 — 2%) and reflectors are not always apparent (Sandmeier & Wenzel, 1990). They also developed an electrical analogue model in which conductive lamellae are isolated in a mafic resistive background but spatially overlapped so as to produce a zig-zag path to current, yielding an effective resistivity as low as that observed. Lamellae need not contain fluids in their model; however, these will be considered brine-saturated here. For micro-pore geometry, quantitative constraints are obtained from associating resistivity data with Archie's law (i-e resistivity vs porosity) and seismic velocities with a pore-geometry "effective-medium" model (i-e velocity vs porosity; e. g. O'Connell & Budiansky, 1974). Thus, lamellae porosity are inferred to range 1 — 3% and connected pore aspect-ratio 0.001 — 0.03 (Hyndman & Shearer, 1989; Cassidy & Ellis, 1991).

From the seismic dissipation viewpoint, Q^{-1} maps from surface-wave inversions generally show little attenuation in the lower crust (Cong & Mitchell, 1988); yet, it is finite and so requires an explanation. It is unlikely that squeeze-flow operates in the lower crust, since significant rock-surface roughness seems absent there. Moreover, as mentioned in chapter-2, the relatively high pore-aspect ratios inferred for lower-crustal lamellae yield extremely high characteristic frequencies for the squirt-flow process, unless saline fluids acquire yet unknown properties at ambient temperature and pressure. Given the fine porous

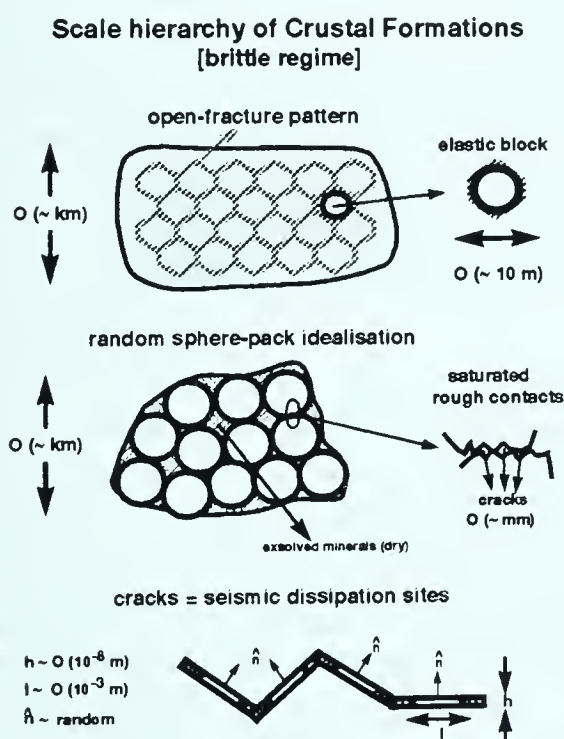


Figure-5.3 Scale hierarchy used for estimating macroscopic squeeze-flow dissipation in brittle crust. Scale is denoted by O (dimension). Top: km-scale formation with fracture porosity portrayed as a pattern of contacting rock-fragments; (right) fragment idealised as an inscribed sphere. Middle: same formation but with spheres randomly packed; packing porosity is filled with exsolved solid-minerals of high compliance; both exsolved materials and sphere are assumed perfectly elastic; (right) mechanical response of packing is controlled by roughness of contacting sphere surfaces. Bottom: squeeze-flow dissipation sites within interspace of rough-contacts; characteristic dissipative cells are ran-

texture expected in a ductile regime, it may appear more appropriate to employ Biot's theory of wave propagation in porous media for estimating dissipation in the lower crust; however, inertial fluid-flow effects, necessary for Biot's mechanism, are unlikely for the inferred lamellae's pore thicknesses (cf. fig-4.3, chapter-4). A good candidate for attenuation in the lower crust, however, is scattering attenuation (Q_{sc}^{-1}) if Merzer & Klemperer's (1992) zig-zag network is accepted, because of the spatial distribution of the lamellae and of their lengthwise dimensions which near seismic wavelengths.

The characterisation of the transitional zone, by its very ambiguous nature, is somewhat difficult. However, it is possible to circumscribe its properties within the framework of an hydrated crust by drawing from Bailey's model (1990) of midcrustal fluid reservoirs in a stable continental environment. Using a magma-percolation model due to McKenzie (1984), Bailey (1990) shows that deep-crustal fluids in the ductile zone must rapidly empty upwards and accumulate at the brittle/ductile boundary within narrow reservoirs generated by hydraulic fracturing. Metamorphic reactions in the cold upper-crust bottom are assumed to seal fluid reservoirs, which can then reside as such over long periods (although adumbrated in his paper, this implies that fluid pressure is near lithostatic for chemical equilibrium to persist, see e. g. Fyfe et al., 1978). Bailey (1990) also points-out that, If the normal state of stress in the brittle crust is compressive and the least principal stress vertical, rock hydraulically fractures sub-horizontally. Given the natural heterogeneity of rock strength, it can be assumed that such reservoirs are structured as intersecting fractures, thereby forming a network whose fracture-porosity is akin to the upper-crust (fig-5.1). Thus, the squeeze-flow mechanism is possible in such configuration, although some recognition that pore-fluid pressure may exceed lithostatic is called for (e. g. if the emptying process is ongoing as in a tectonically active zone).

Finally, it is important to relate the 400 ± 50 °C marker to the thermodynamic properties of H₂O-rich fluids. Although the equation of state of pure H₂O sets the critical point at 374 °C and 22.1 MPa, addition of realistic amounts of NaCl generates loci of critical points on a phase diagram, hence bringing to higher temperatures and pressures the demarcation between liquid and gas state (Bodnar & Costain, 1991). In this work, I describe the role of nominally saline-fluids in seismic dissipation for thermodynamic conditions for which these fluids are in the liquid-state, i-e for temperatures which may exceed that of the critical point of pure H₂O.

5.2 Wave propagation and dissipation in fragmented crust

Adopting the view that the brittle crust is fragmented requires an appropriate model for wave propagation and dissipation, with a clear parameterisation of characteristic material and mechanical properties. If a network of large-scale rock fragments can be approximated by a randomly ordered packing of like-spheres, the effective elastic moduli model of Digby (1981) for unsaturated porous

granular rocks provides a basis for establishing relationships between elastic wave-velocities and cracked material properties. From these, dissipation can be incorporated by invoking the correspondence principle of linear viscoelasticity, thereby attributing a real and imaginary part to material properties from which Q^{-1} and associated dispersion can be defined (cf. Murphy et al. 1986). Prior to doing so, it is best to understand the mechanical behaviour of dry porous rock first.

Digby's (1981) model is developed solely on stress equilibrium conditions at sphere contacts, without assumptions about pore geometry or pore-space connectivity. This is an advantage over other effective-medium techniques based on perturbation of the elastic energy-field by sequential insertion of cracks of assumed geometries (e. g. O'Connell & Budiansky, 1974, 1977), which lead to conflicting results regarding nearby crack interactions (Heyney & Pomphrey, 1982) and to parameters, such as preferential crack-density, which are difficult to confidently estimate in practice (Bradley, 1976).

In Digby's model, the effective elastic moduli depend on the nominal mechanical stiffness of contacting spheres, whose surfaces are assumed smooth, and sphere size is much smaller than wavelength, which implies negligible scattering. Winkler (1983) inverted Digby's effective moduli expressions to give contact stiffnesses in terms of compressional and shear velocities. An important qualification of Winkler's (1983) stiffness relations was recently put forth by Palciauskas (1992) who examined the effect of sphere-surface roughness on contact stiffness by applying stress-displacement laws proposed by Yoshioka & Scholz (1989a) for a rough-walled rock-fracture under combined shearing and compression.

Combining these results, the wave-velocity/contact-stiffness relations for a dry aggregate of randomly packed spheres can be written

$$\bar{v}_p^2 = (M + \frac{4}{3}N) / \rho_c = \frac{3c}{20\pi\rho_s R_s} \left[D_n + \frac{2}{3}D_t \right] \quad [5.1]$$

$$\bar{v}_s^2 = N / \rho_c = \frac{c}{20\pi\rho_s R_s} \left[D_n + \frac{3}{2}D_t \right] \quad [5.2]$$

where $\bar{v}_p$ and $\bar{v}_s$ are effective compressional (P) and shear (S) wave velocities; M and N effective bulk and shear modulus of the dry aggregate, respectively; c average number of contact points between neighbouring spheres (i-e the "coordination number"); R_s and ρ_s sphere radius and density; ρ_c aggregate density [i-e $(1 - \phi_p) \rho_s$, ϕ_p packing porosity]; and D_n and D_t are normal (n) and tangential (t) contact stiffness. The explicit expressions for D_n and D_t are

$$D_n = \frac{12\pi R_s \rho_s}{c} \left[\bar{v}_p^2 - \frac{4}{3}\bar{v}_s^2 \right] \quad [5.3]$$

$$D_t = \frac{24\pi R_s \rho_s}{c} \left[\bar{v}_s^2 - \frac{1}{3}\bar{v}_p^2 \right] \quad [5.4]$$

It is seen from Eq-5.1 $\rightarrow$ Eq-5.4 that neither velocity nor contact stiffness depends on porosity of the sphere-pack. Moreover, since Eq-5.1 and 5.2 take their source

from stress equilibrium conditions, these relations are independent of the specific micromechanical model of contact stiffness D_n or D_t . With respect to the aim of this chapter, Eq-5.3 and 5.4 are enticing in that the real part of the complex squeeze-flow crack-stiffness due to peripheral solid-solid contacts, disregarded in chapter-4, can be estimated from velocity measurements in dry rock.

An enlightening relation is the ratio

$$\left(\frac{\bar{v}_p}{\bar{v}_s}\right)^2 = \frac{6(D_n/D_t) + 4}{2(D_n/D_t) + 3} \quad [5.5]$$

which entails constraints on the applicability of the sphere-pack model to wave-propagation in rocks *in situ*, since $(\bar{v}_p/\bar{v}_s)^2$ is intimately linked to effective Poisson's ratio $\bar{\nu}$, an important material property sensitive to cracks and their degree of saturation (Nur & Simmons, 1969; Spencer & Nur, 1976). Eq-5.5 shows that, as D_n/D_t varies from one to infinity, $(\bar{v}_p/\bar{v}_s)^2$ varies from 2 to 3. $(\bar{v}_p/\bar{v}_s)^2$ is related to $\bar{\nu}$ by

$$\bar{\nu} = \frac{(\bar{v}_p/\bar{v}_s)^2 - 2}{2(\bar{v}_p/\bar{v}_s)^2 - 2} \quad [5.6]$$

and so the application of Eq-5.1 and 5.2 is limited to dry porous rock with effective Poisson's ratio bounded by $0 \leq \bar{\nu} \leq 0.25$.

Winkler (1983) calculated D_n and D_t using specific contact-stiffness models for smooth sphere-surfaces based on Digby's (1981) work and the classical Hertz-Mindlin theory (Mindlin, 1949), and tested Eq-5.5 against $\bar{v}_p$ and $\bar{v}_s$ measurements in dry monomineralic granular rocks and unconsolidated glass beads under increasing confining pressure (up to 73 MPa). Although both observed and calculated $\bar{v}_p/\bar{v}_s$ ratios were found approximately constant at all pressures (an interesting result in itself since $\bar{v}_p, \bar{v}_s$ clearly depend on composition and pressure), they significantly differed in magnitude.

Palciauskas (1992) expanded on Winkler's work by using a contact-stiffness model for rough surfaces developed by Yoshioka & Scholz (1989a), from which he deduced the expression

$$D_n/D_t \equiv 1.40 \left(\frac{2 - \nu}{1 - \nu^2} \right) \quad [5.7]$$

where ν is the intrinsic Poisson's ratio of the contacting mineral, typically ranging $0.08 \leq \nu \leq 0.40$. Despite this very wide range of values, insertion of Eq-5.7 in Eq-5.5 yields a nearly constant $\bar{v}_p/\bar{v}_s \sim 1.55$, in acceptable agreement with observations. Although Eq-5.7 is erroneous (it is corrected in the next section), Palciauskas' result underscores the important observation that, although measurements of $\bar{v}_p$ and $\bar{v}_s$ in dry monomineralic granular rocks vary widely with composition, porosity, and confining pressure, the ratio $\bar{v}_p/\bar{v}_s$ remains constant and so practically insensitive to these attributes. This result is useful for investigating the applicability of Eq-5.1 and Eq-5.2 to crustal-scale rock fragments with

rough contacts.

5.2.1 Application to crustal-scale fractured rock-formations

Taken at face value, Palciauskas' (1992) result of a constant $\bar{v}_p/\bar{v}_s \sim 1.55$ indicates that the rough-surface sphere-pack model is valid inasmuch the material is monomineralic and has an effective Poisson's ratio $\bar{\nu} \sim 0.14$, at least up to confining pressure $p_c \sim 73$ MPa. As such, it seems futile to even consider applying this model since, in the Earth's crust, collocated $\bar{v}_p$ and $\bar{v}_s$ observations generally amount to much higher $\bar{\nu}$'s ($0.14 \leq \bar{\nu} \leq 0.40$), p_c has a nominal gradient of 26 MPa/km which rapidly yields crushing pressures for soft grain contacts, and, of course, *in situ* rocks are polycrystalline. Nevertheless, application of the sphere-pack velocity/stiffness relations to crustal-scale fractured formations can be justified by going back to the original contact stiffness expressions of Yoshioka & Scholz (1989a) which have led to Eq-5.7. The idea, here, is to establish an applicability criterion by estimating the effect of polycrystallinity and modelling limits on D_n/D_t and examine, via Eq-5.5, the range of values shared with $\bar{v}_p/\bar{v}_s$ observations from areas of the crust which can be considered fractured and dry. In the process, the error made by Palciauskas (1992) becomes apparent.

Yoshioka & Scholz (1989a) developed a contact theory of two elastic rough surfaces under concurrent compression and shearing, without slip and in which normal (σ_n) and tangential (τ) stresses are related to the normal (δ) and shear (δ_s) displacements of statistically distributed hemispherical asperities in Hertzian contact. The roughness of each surface is characterised by a probability density function of asperity topography. From these stress-displacement relations follow definitions of stiffnesses D_n and D_t , succinctly written:

$$D_n \equiv \frac{\partial \sigma_n}{\partial \delta} = \frac{8}{3} \pi \bar{E} \Xi F(z_a, \frac{3}{2}) \quad [5.8]$$

$$D_t \equiv \frac{\partial \tau}{\partial \delta_s} = \frac{16}{3} \sqrt{\pi} \frac{\Gamma(5/6)}{\Gamma(8/6)} \bar{G} \Xi F(z_a, \frac{3}{2}) \quad [5.9]$$

where $\bar{E}$ and $\bar{G}$ are "reduced" Young's and shear modulus of the surface pair, respectively; Ξ a factor comprising hemispherical geometry parameters and asperity densities; Γ Gamma function; z_a an overlap distance parameterising the closure ($\equiv \delta$) of the two surfaces; and $F(z_a, 3/2)$ is a double integral over z_a of the two probability density functions characterising roughness.

The reduced moduli $\bar{E}$ and $\bar{G}$ are defined by

$$\frac{1}{\bar{E}} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \quad [5.10]$$

$$\frac{1}{\bar{G}} = \frac{2 - \nu_1}{G_1} + \frac{2 - \nu_2}{G_2} \quad [5.11]$$

where the labelling specifies the intrinsic material properties (Poisson's ratio ν_i , Young's modulus E_i , shear modulus G_i) of the first and second surface. Dividing Eq-5.8 by Eq-5.9 yields

$$D_n/D_t = \frac{1}{2} \sqrt{\pi} \frac{\Gamma(8/6)}{\Gamma(5/6)} \frac{\bar{E}}{\bar{G}} \quad [5.12]$$

Supposing that all asperities are materially homogeneous, $E_1 = E_2 = E$; $G_1 = G_2 = G$; $\nu_1 = \nu_2 = \nu$; and Eq-5.12, with the numerical factor evaluated, reduces to

$$D_n/D_t \cong 1.40 (1 + \nu) \left(\frac{2 - \nu}{1 - \nu^2} \right) \quad [5.13]$$

where I made use of the correct relation (Jaeger & Cook, 1971, p. 104)

$$E = 2G (1 + \nu) \quad [5.14]$$

Palciauskas (1992) used a false relation between E and G , which explains the additional factor $(1 + \nu)$ in Eq-5.13, absent in Eq-5.7. Heuristically, this extra factor does not invalidate Palciauskas' analysis; whilst giving a variable $\bar{v}_p / \bar{v}_s$ ratio, the corrected result agrees much better with his observations. The ensuing bounds on effective Poisson's ratio become $0.15 \leq \bar{\nu} \leq 0.17$, which covers the lower end of the range for crustal values.

The effect of dissimilar mineralogy on D_n / D_t poses no problem since the intrinsic material properties in Eq-5.10 and Eq-5.11 can be ascribed to those of intact rocks. Hence, provided the two contacting rock-surfaces are materially identical, Eq-5.13 applies as it stands. Tabulated values of Poisson's ratios for crustal rock-samples (Angenheister, 1982) also range $0.08 \leq \nu < 0.40$; therefore, $2.92 \leq D_n / D_t \leq 3.73$; $1.56 \leq \bar{v}_p / \bar{v}_s \leq 1.59$, assuming Yoshioka & Scholz' (1989a) contact-stiffness relations exact. In a companion paper, Yoshioka & Scholz (1989b) tested their contact theory against stiffness observations obtained from curves of joint closure versus applied stress; data points fitted the theory, roughly within $\pm 5\%$. Taking this latter value as error bounds on D_n / D_t , a conservative range for the velocity ratio is $1.50 \leq \bar{v}_p / \bar{v}_s \leq 1.65$, to which corresponds an effective Poisson's ratio in the range $0.10 \leq \bar{\nu} \leq 0.21$.

Figure-5.4 shows a diagram of $\bar{v}_p / \bar{v}_s$ versus $\bar{\nu}$ obtained from inserting Eq-5.13 for crustal rock-sample values in Eq-5.5 and Eq-5.6 (straight line with bounds), along with representative values for the upper crust (curve, from Eq-5.6). The domain of applicability of the random sphere-pack model to crustal formations appears as a box overlapping crustal observations, as shown. Several field studies of collocated shear and compressional velocities pertaining to the upper crust have reported low $\bar{v}_p / \bar{v}_s$ ratios which do fall within the applicability domain of the sphere-pack model (e.g. Assumpção & Bamford, 1978; Nicholson & Simpson, 1985; Sandmeier & Wenzel, 1990; Thurber & Atre, 1993). Although, by themselves, such seismic data cannot ensure that low $\bar{v}_p / \bar{v}_s$ ratios result from dry cracks and not from an over-abundance of particularly compliant quartz-rich rocks or from wet cracks in peculiar geometries, the uncertainty can be

reduced by ancillary constraints.

Laboratory data of $\bar{v}_p / \bar{v}_s$ in dry and saturated cracked-rocks indicate that, in general, dry rocks exhibit lower ratios than wet ones (Nur & Simmons, 1969; Spencer & Nur, 1976). With regards to the very existence of cracks in the upper crust, their presence in various degrees of saturation and interconnectivity is demonstrated from deep-drilling results. Moreover, inasmuch as rock-fracture walls are rough, dry open space can exist in the brittle crust down to at least 18 km before ambient pressures approach asperity strength, as shown by Scholz (1988). Accepting the pervasiveness of cracks in the upper crust, constraints on their degree of water-saturation are provided by electrical resistivity profiles, with high resistivity suggestive of dryness.

Although high resistivity can also result from a water-film network interweaved with a poorly conductive CO_2 fluid phase, at upper crustal conditions CO_2 likely is in the gas state (Bodnar & Costain, 1991) and so imparts to the rock frame a high compliance, approaching that of a dry frame. Hence, the combination of high resistivity and low $\bar{v}_p / \bar{v}_s$ ratios in the upper crust can be assumed representative of crustal formations which are fractured and dry. Therefore, the effective contact-stiffness of formations with such attributes can be calculated from velocity measurements in such areas using Eq-5.3 and Eq-5.4, provided other parameters can be estimated.

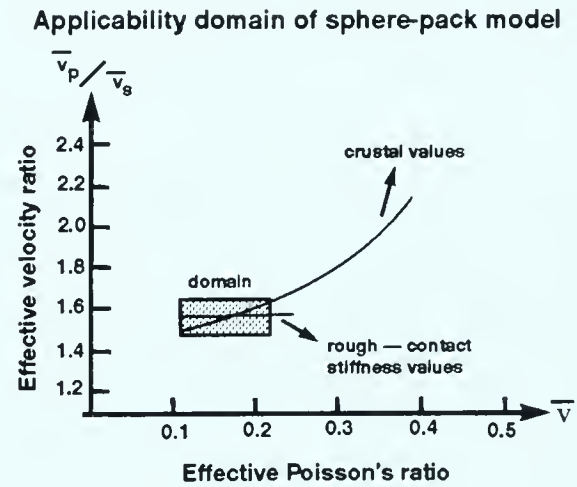


Figure-5.4 Relationship between $\bar{v}_p / \bar{v}_s$ and $\bar{\nu}$ for packing of rough-surface spheres in mechanical contact (boxed-in horizontal line) and for range of velocity ratios measured in the field (curve). Sphere-pack model is applicable for $\bar{v}_p / \bar{v}_s$ data confined to the shaded part of the graph. Observational error on $\bar{\nu}$ (not shown) is at least ± 0.02 .

5.2.2 Adding fluid and dissipation/dispersion by squeeze-flow

D_n and D_t from Eq-5.3 and Eq-5.4 provide estimates for the effective elastic stiffness of a dry rock-formation which, although pervaded by microscopic fracture-inter-space cracks, behaves as an homogeneous material at the macroscopic scale. As such, these contact-stiffnesses account for the fact that joints and fractures in a rock

body augment its macroscopic compliance relative to intact conditions. This compliance is reduced by adding fluid in the fracture interspace and becomes frequency-dependent. The dynamic nature of the compliance of fractured rock saturated with a fluid phase can be described within the framework of linear viscoelasticity by adding a complex term parameterising the effect of fluids. This procedure was used by Murphy et al. (1986) for computing "squirt-flow" Q^{-1} from the ratio of real-to-imaginary part of an effective complex modulus (inverse of compliance) modelled for granular rock and can be applied to squeeze-flow in fractured rock using the crack stiffness developed in chapter-4.

A general definition of the macroscopic dynamical (complex) modulus for sheared fractured rock can be written $\bar{N}(\omega) \equiv N_{\text{real}}(\omega) + i N_{\text{imag}}(\omega)$, in which real and imaginary parts can be specialised to the sphere-pack model, by way of Eq-5.2 and combined with the squeeze-flow model using the complex stiffness $s(\omega)$ of a crack typical of rough fractures, as defined in chapter-4. Thus, following Murphy et al. (1986), the macroscopic $Q^{-1}(\omega)$ response and associated dispersion $v(\omega)$ caused by squeeze-flow in sheared fractured rock can be defined as:

$$Q^{-1}(\omega) \equiv \frac{N_{\text{imag}}(\omega)}{N_{\text{real}}(\omega_{\text{ref}})} = \frac{\Phi_{\text{Macro}} [s_{\text{imag}}(\omega)]_{\text{micro}}}{(D_n + \frac{3}{2}D_t)_{\text{Macro}} + \Phi_{\text{Macro}} [s_{\text{real}}(\omega_{\text{ref}})]_{\text{micro}}} \quad [5.15]$$

$$v(\omega) \equiv \frac{N_{\text{real}}(\omega)}{N_{\text{real}}(\omega_{\text{ref}})} = \frac{(D_n + \frac{3}{2}D_t)_{\text{Macro}} + \Phi_{\text{Macro}} [s_{\text{real}}(\omega)]_{\text{micro}}}{(D_n + \frac{3}{2}D_t)_{\text{Macro}} + \Phi_{\text{Macro}} [s_{\text{real}}(\omega_{\text{ref}})]_{\text{micro}}} \quad [5.16]$$

where ω_{ref} is a reference frequency taken as $\omega \rightarrow 0$; Φ is the porosity associated with dissipative cracks in the fracture interspace; and the labelling "Macro" and "micro" reminds of the scale of the corresponding property.

Eq-5.15 and Eq-5.16 are similar to those of Murphy et al. (1986), except for an important difference: the fluid effect is weighted by the macroscopic porosity Φ and not by the parameter which they define as "the number density of highly dissipative contacts". Their weight factor, which is poorly constrained, is identical to the "crack-density parameter" ϵ_c originating from effective-medium models based on O'Connell & Budiansky's (1974) approach; it is defined in terms of crack aspect-ratio and so represents only that part of the total porosity occupied by cracks of a common aspect-ratio. Although ϵ_c estimates for crustal formations (typically $0.02 \leq \epsilon_c \leq 0.07$) have been reported from observations of seismic anisotropy (Crampin, 1987), it seems perilous to connect them with dissipation by squeeze-flow since, in the anisotropy formalism, cracks are assumed ellipsoidal and isolated and so do not allow significant viscous flow.

Using macroscopic porosity for weighting fluid effects

on mechanical moduli is not without difficulties either, particularly when considering that, apart from having to relate Φ to actual *loci* of squeeze-flow, a portion of natural fracture interspace is fractal; therefore, devoid of a "characteristic" dimension proper. Squeeze-flow is postulated to occur within a critical neck along fluid paths in between rough-walled fractures; despite the fractal geometry of individual walls, this critical neck is amenable to a single dimensional characterisation, as shown by Scholz (1988). However, in chapter-4, I modelled the characteristic crack associated with the critical neck as a parallel-plate disk in contact with a "porous ring", in which ring porosity ϕ is a sensitive *microscopic* parameter of dissipation. Hence, there are three questions to consider: 1) how is Φ estimated for a medium pervaded by a network of rough-walled fractures?; 2) what is the link between macroscopic porosity Φ and cracks conducive to squeeze-flow?; and 3) what is the relationship between Φ and microscopic porosity ϕ ?

These questions can be answered by interpreting the characterisation of natural fracture geometry recently developed in fracture-mechanics and cognate fields (Brown & Scholz, 1985; Brown, 1986; Pyrak-Nolte et al., 1988). Figure-5.5 illustrates the essential concept behind rough-fracture characterisation: a sketch of the power spectral density of asperity height $G(\kappa)$ versus spatial wavenumber $\kappa = 2\pi/\lambda$ (λ = length or distance), obtained in practice from profilometer data. By definition, $G(\kappa)$ is the square of the normalised Fourier transform of the autocorrelation function and so expresses the degree with which asperities of equal heights are correlated over distance λ ; the area under $G(\kappa)$ is a measure of the variance of the profile.

The sketch is common to all rough-surfaces measured, regardless of rock type. It is seen that for a single rough surface the variance of height increases with asperity length. In this situation, asperity height and length are correlated, with their ratio grossly scale-invariant; therefore, asperity dimensions cannot be "averaged" in any meaningful way. When two such surfaces are joined, however, the larger asperities come into contact, resulting in two important features: 1) a cut-off wavenumber below which there is no power, corresponding to occluded zones; and 2) a characteristic distance λ_c corresponding to the maximal power, about which the nature of void space differs (shaded strip). This λ_c and associated height quantify the dimension of a characteristic crack which, by virtue of its corresponding maximal power, predominates the void space spatial distribution. Its physical meaning is that it represents both the smallest asperity size and largest asperity spacing (Scholz, 1988).

Roughness measurements of contacted fractures (e.g. Brown, 1986) show an approximately constant power for $\lambda > \lambda_c$; since variance is constant there, averaging of asperity dimensions is meaningful. Moreover, that the power level is finite ensures that there are some relatively large flow conduits which bring fluid to the critical necks throughout the fracture interspace. Thus, such void geometry can be considered Euclidean. For $\lambda < \lambda_c$, however, void space remains fractal and there are no meaningful averaging of flow parameters. This interpretation of surface-roughness spectra inspires the fracture interspace

representation shown in Figure-5.6, in relation to the microgeometrical elements of squeeze-flow.

Characterisation of fracture interspace

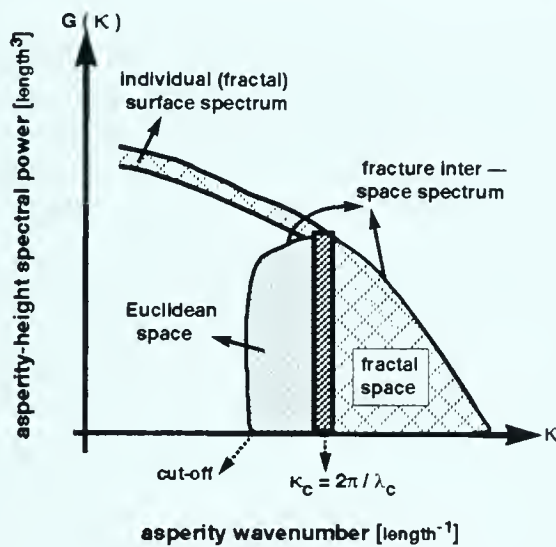


Figure-5.5 Composite sketch of rock-surface roughness power spectrum characterising fracture interspace, after Brown & Scholz (1985) and Brown (1986). Single surfaces are fractal. Joined surfaces separate void-space into an Euclidean part and a fractal part. λ_c is both the largest asperity spacing and the smallest asperity contact (Scholz, 1988). Squeeze-flow is located in cracks of the order of λ_c

Rough Fracture Interspace Representation

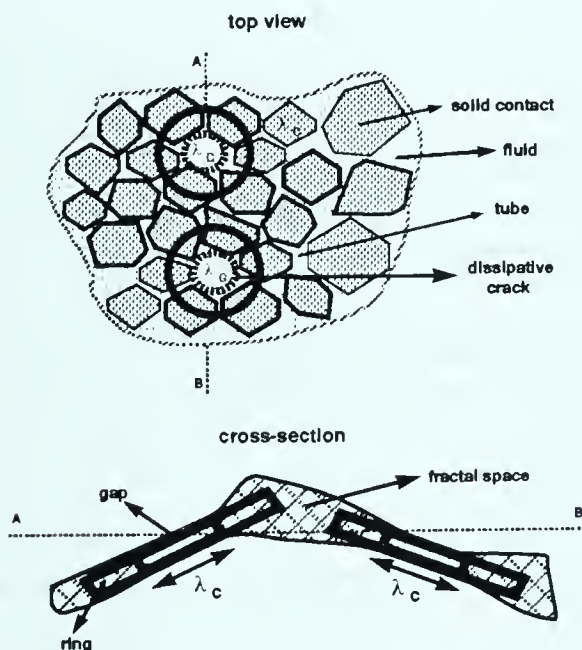


Figure-5.6 Fracture interspace representation in harmony with the interpretation of rough-fracture interspace power spectra (fig-5.5). Larger tubes belong to Euclidean space, smaller ones surrounding squeeze-flow gaps to fractal space.

With respect to estimating macroscopic Φ , it is reasonable to regard the fracture network associated with rough-

contacts in the sphere-pack model as being composed of randomly oriented fractures. The medium can then be considered isotropic and permeable, with its macroscopic permeability K a function of fracture porosity [i.e. $K = K(\Phi)$]. Pyrak-Nolte et al. (1988) simulated fluid-flow within a rough fracture using percolation theory and showed that K is governed by "the smallest aperture along the path of maximal aperture" in the fracture interspace (hence providing support for the equivalent channel model, cf. chapter-2). Logically, maximal aperture paths belong to the Euclidean part of the interspace and minimal apertures along these paths correspond to critical necks, that is to characteristic cracks. Since the latter dominate the interspace spectrum, it is reasonable to expect that permeability measurements mainly reflect the porosity associated with characteristic cracks. Therefore, porosity Φ can be estimated from crustal permeability estimates, provided the relationship $K(\Phi)$ is specified. Many such relationships have been proposed (Scheidtger, 1974) and most for high porosity materials, unlike the low porosity typical of *in situ* crustal rocks. Nevertheless, the empirical relation used by McKenzie (1984) and Bailey (1990) for modelling crustal-fluid flow, based on permeability/porosity data for low-porosity rocks, can be used since it is appropriate for mm-scale pore-length which also corresponds to the length of characteristic cracks (chapter-2). It is written here as:

$$K = (\lambda_c^2 \Phi^3) / 1000 \quad [5.17]$$

There remains the problem of elucidating the meaning of microscopic porosity ϕ and its link with Φ . This aspect is best discussed in terms of their associated permeabilities, with k that corresponding to ϕ . Within the framework of the squeeze-flow dissipative cell model (chapter-4), the parallel-plate disk part shares its radial periphery with the inner radial surface of the porous-ring part whose finer structure, as suggested by fig-5.6, is fractal. ϕ characterises the porosity of both this common boundary and that of the ring itself which has a finite size; as such, it is clearly a microscopic parameter. Of course, this view of interspace geometry is highly idealised but little else can be done if the problem is to be kept mathematically tractable; at the same time though, it remains conceptually consistent with permeable media physics.

Some work has been done on characterising permeability/porosity relations for fractal porous rocks. Of particular interest is a recent paper by Hansen & Muller (1992) who successfully fitted (k, ϕ) data for rock-samples of low permeabilities and known to be fractal. Their modelling is based on mean field theory (Kirkpatrick, 1973), from which the effective permeability of a random network of fluid conductors can be expressed as an integral over a probability distribution function for the microscopic conductances. Using a two-level uniform distribution comprising porosity and a range of permeabilities as variables, they show that the effective permeability k and porosity ϕ of a fractal pore-space obey the implicit relation:

$$\frac{1}{1 + a_n} \frac{1}{k} = \left[\frac{(1 - \phi)}{\Delta k_d} \text{Log} \left(\frac{\Delta k_d + a_n k}{a_n k} \right) + \frac{\phi}{\Delta k_c} \text{Log} \left(\frac{k_c + \Delta k_c + a_n k}{k_c + a_n k} \right) \right] \quad [5.18]$$

where Δk_d is the range of permeabilities for those parts of the structure which are disconnected from the main conductive field; and $k_c + \Delta k_c$ bounds the range of permeabilities from highest to lowest possible values for the main conductive field. a_n is a connectivity parameter defined as $a_n = (n_b/2) - 1$, with n_b the number of bonds joining at each node of the representative network. Observations, albeit scarce, constrain a_n to $6 \leq a_n \leq 12$, with $a_n = 9$ yielding the best data-fitting.

Following the line of thought behind fig-5.5, the range $k_c + \Delta k_c$ can be estimated by taking maximal permeability as that for the characteristic crack and minimal permeability as that for the smallest void capable of accommodating a few liquid monolayers, for example via equivalent channel estimates which may also be applied to Δk_d (e. g. Paterson, 1983). Thus, both K and k are linked, as are their associated Φ and ϕ . Accepting the experimentally constrained $a_n = 9$, Eq-5.18 can be numerically solved for both k and ϕ , thereby providing estimates for the microscopic permeability and porosity of the ring part of the squeeze-flow cell.

5.3 Attenuation models in a stratified hydrated crust

The macroscopic shear $Q^{-1}(\omega)$ and $v(\omega)$ response to squeeze-flow in hydrated brittle crust can now be computed using Eq-5.15 & Eq-5.16, along with estimates of representative macroscopic properties for the structure and microscopic properties for the microstructure, as depth increases. The fundamental observables are P and S -wave velocity measurements from a dry and fractured portion of the crust and large-scale permeability estimates from reservoir-induced seismicity (RIS) manifestations. From these, rough-fracture contact-stiffnesses can be computed from Eq-5.3/5.4 and fracture-porosity from Eq-5.17. There are four required macroscopic properties: sphere radius R_s and material density ρ_s ; random sphere-pack coordination number c ; and porosity Φ . The required microscopic properties are (Eq-4.44, 4.45, chapter-4): disk-gap radius a ($\equiv \lambda_c$) and thickness h_0 ; ring radius b (passive) or R (active); compressibility of fluid β and solid α ; fluid viscosity μ ; and ring porosity ϕ and permeability k .

In order to illustrate variation with depth, I divided the upper-crust in three layers, each 5 km thick. Estimated macroscopic parameters are shown in Table-5.1. I varied the coordination number c from 6 to 12, guided by Smith, Foote & Busang's (1929) measurements for tight random sphere-pack. The increase of c with depth is suggested by Nur's (1982) fracturing models for fracture-controlled lineaments. With this line of reasoning, I also varied

sphere radius R_s from 20 m to 40 m, as suggested by some field studies. I computed contact-stiffness D_n and D_t from Eq-5.3/5.4 using densities characteristic of felsic rocks (tabulated as a function of pressure in Angenheister, 1982) and velocity magnitudes respecting the applicability criterion of fig-5.4, in harmony with field data from areas of high electrical resistivity or for which the presence of gas can be inferred (Assumpção & Bamford, 1978; Nicholson & Simpson, 1985; Sandmeier & Wenzel, 1990; Thurber & Atre, 1993). Macroscopic porosity Φ was obtained from Eq-5.17, using crustal permeability estimates from reservoir-induced seismicity (RIS) reported by Brace (1984). Here too, I varied K with depth as suggested by Talwani & Acree's (1985) compilation of documented RIS occurrences.

Table-5.2 shows estimates of the microscopic parameters. k and ϕ were obtained by numerically solving Eq-5.18 with $a_n = 9$. Typically, $k < K$ as expected, whilst ϕ remains relatively constant [see $k(\phi)$ plots in Hansen & Muller, 1992]. Gap radii are constrained by roughness measurements of "fresh" contacting surfaces (Scholz, 1988) and gap thicknesses by allowing for mineral coatings at characteristic crack boundaries, as explained in section-2.3.2. To satisfy the physical meaning of λ_c and associated geometrical constraints, passive ring radius b was set at 1/2 of gap radius, active ring radius R at twice the gap radius. Ring solid compressibility α is estimated from Angenheister's (1982) tables at the specified pressure. Fluid viscosity μ is that of liquid-water at 0.5 M of NaCl, as estimated from Haar, Gallager & Kell (1984) and Kestin et al., (1981), assuming a temperature gradient of 13 °C/km (Shankland & Ander, 1983) and a pressure gradient of 15 MPa/km (midway between hydrostatic and lithostatic). Fluid compressibility β was computed from its thermodynamic definition, using pure-water densities at appropriate temperatures and pressures tabulated in Haar et al. (1984).

All calculations and graphics were performed on a NeXT™ machine, with the Mathematica™ software (Wolfram, 1988). Except otherwise indicated, results are plotted on the same log-log scale as seismic measurements in the field, i-e with attenuation magnitude varying from 10^{-4} to 10^{-1} versus frequency varying from 10^{-2} to 10^2 Hz. The crustal seismic band is defined as 0.08 — 25 Hz.

Table 5.1: Squeeze-flow Q^{-1} macroscopic parameters [upper crust]

Depth-range → [km]	0 — 5	5 — 10	10 — 15
Permeability K [10^{-15} m^2]	100	1	.1
P-wave velocity $\bar{v}_p$ [m / s]	5700	6000	6300

Table 5.1: Squeeze-flow Q^{-1} macroscopic parameters [upper crust]

Depth-range → [km]	0 — 5	5 — 10	10 — 15
S-wave velocity $\bar{v}_s$ [m / s]	3560	3675	3820
Coordination number c [.]	6	9	12
Sphere density ρ_s [kg / m ³]	2600	2630	2650
Sphere radius R_s [m]	20	30	40
Macroscopic porosity Φ [.]	0.016	0.010	0.005
Normal contact-stiffness D_n [10 ¹² Nt / m]	5.1	6.0	7.0
Tangential contact-stiffness D_t [10 ¹² Nt / m]	1.2	1.0	0.9

Nota: macroscopic porosity Φ is related to microscopic parameter a ($\equiv \lambda_c$).

Figure-5.7 shows squeeze-flow dissipation (Q^{-1}) and associated dispersion $v(\omega)$ (normalised to low-frequency limit) at depth-range 5 — 10 km, for both passive and active ring models of chapter-4. As expected from linear viscoelasticity and the simple scaling implicit in the definition of Q^{-1} used here, the frequency signature of dissipation is mainly controlled by the microscopic squeeze-flow mechanism, whereas its magnitude is primarily controlled by macroscopic parameters, in particular porosity Φ and rough-contact stiffnesses. As in chapter-4, squeeze-flow with active ring is larger than for the passive ring. The two models predict Q^{-1} 's with different asymptotic behavior at high-frequencies (after peak frequency), giving a negative slope of 1/2 for the passive ring, 2/3 for the active ring. At low-frequencies Q^{-1} linearly increases for both. Both models show peak Q^{-1} 's, well within the crustal-seismic band and of significant magnitude. In harmony with linear viscoelasticity, the associated velocity dispersion varies from a minimum at low-frequency to a maximum at high frequency, with a magnitude of the order of one per cent. Since dissipation and dispersion are intimately linked (one is roughly the slope of the other), only $Q^{-1}(\omega)$ curves are shown hereon. Moreover, because squeeze-flow with active ring is more consistent with the premiss that localised differential stresses drive the mechanism, only active ring results are shown, the relationship between both ring models being the same than depicted in fig-5.7 for all other conditions

considered.

Table 5.2: Squeeze-flow Q^{-1} microscopic parameters [upper crust]

Depth-range → [km]	0 — 5	5 — 10	10 — 15
Pressure P [MPa]	75	150	225
Temperature T [°C]	65	130	200
Fluid viscosity μ [10 ⁻⁶ Pa s]	700	400	200
Fluid compressibility β [10 ⁻¹¹ Pa ⁻¹]	39	36	37
Solid compressibility α [10 ⁻¹¹ Pa ⁻¹]	2.5	2.0	1.7
Gap thickness h_0 [10 ⁻⁹ m]	20	2	1
Gap radius a [10 ⁻³ m]	5	1	1
Passive ring radius b [10 ⁻³ m]	2.5	0.5	0.5
Active ring radius R [10 ⁻³ m]	10	2	2
Active ring permeability k [10 ⁻¹⁵ m ²]	10	.1	0.01
ring porosity ϕ [.]	0.20	0.15	0.10

Nota: microscopic parameters k and ϕ are related to macroscopic permeability K .

For comparing dissipation by squeeze-flow, which operates in critical necks, to dissipation by relative solid-frame/fluid motion, which may be significant in the large pores of Euclidean fracture-space, I computed shear Q^{-1} using Biot's theory as formulated by Géli et al. (1987) for the depth-range 5 — 10 km. Biot's theory also predicts a critical frequency at which both viscous and inertial effects combine to produce maximal dissipation. The result is shown in figure-5.8, where it is seen that Biot's mechanism is irrelevant to crustal seismics. Although several authors have made this observation (Géli et al. 1987; Rasofoloson, 1991), Toksöz et al. (1990) still

claim that part of crustal attenuation is due to Biot's mechanism; therefore, the above statement is not totally vacuous.

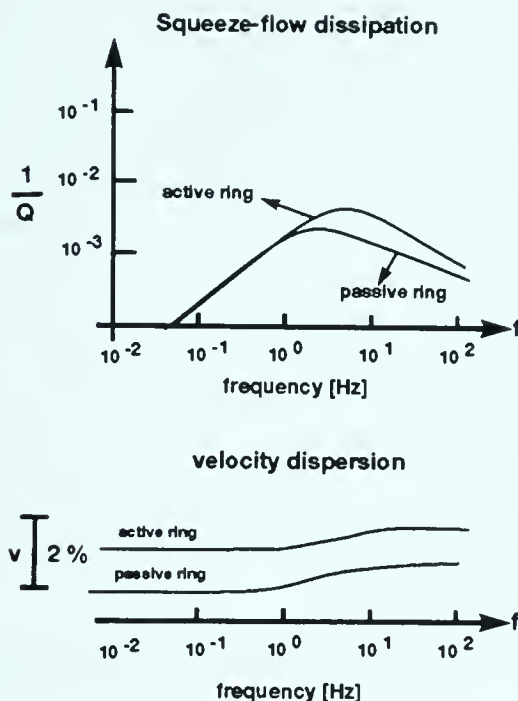


Figure-5.7 Predicted squeeze-flow dissipative $Q^{-1}(\omega)$ and associated dispersion $v(\omega)$ in the depth-range 5 — 10 km. Macroscopic and microscopic properties are listed in table-5.1 and 5.2, respectively. For active ring dissipative cell, $Q^{-1}(\omega)$ decreases at high-frequencies with a slope of $-2/3$, for passive ring with $-1/2$. $Q^{-1}(\omega)$ for both ring models linearly increases at low-frequencies. Dispersion is of the order of one% (2% bar indicates scale). Crustal-seismic band of interest is 0.08 — 25 Hz.

signature of Q^{-1} , the conclusions reached in chapter-4 regarding parameter sensitivity integrally apply here. Macroscopic Q^{-1} magnitude is affected by uncertainties in the parameters of Table-5.1; however, these are not "free parameters" per se since they are subjected to relatively narrow constraints from field-based manifestations of crustal hydraulic properties. Thus, it can be expected that macroscopic $Q^{-1}(\omega)$ retains its robust frequency character in the upper crust.

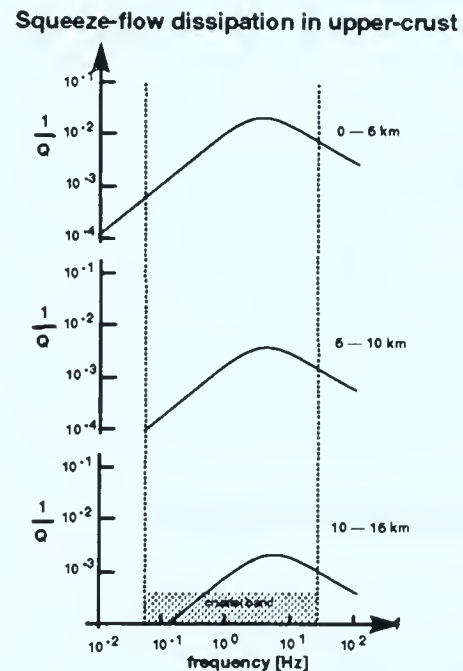


Figure-5.9 Depth-distribution of squeeze-flow dissipation (active ring) in upper crust. $Q^{-1}(\omega)$ decreases with depth whilst its peak value remains within the crustal-seismic band.

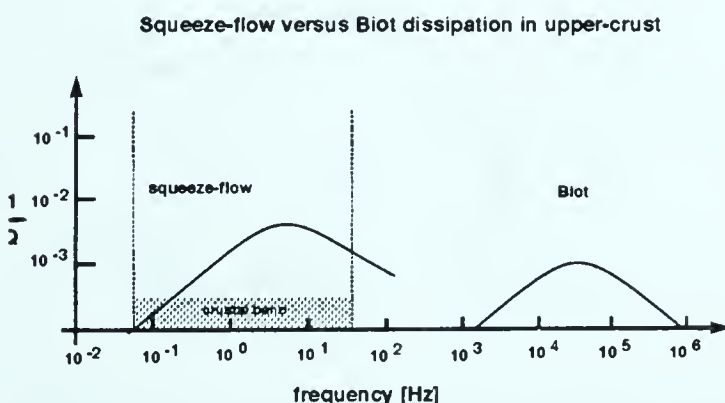


Figure-5.8 Comparison of active-ring cell squeeze-flow with Biot's bulk-flow mechanism, for the depth range 5 — 10 km. Biot $Q^{-1}(\omega)$ linearly increases at low-frequencies and decreases at high-frequencies with a slope of $-1/2$; intermediate peak marks the point of highest values of both viscous and inertial flow.

Figure-5.9 shows squeeze-flow Q^{-1} for the three parts of the upper crust parameterised in Table-5.1 and Table 5.2. It is seen that peak values remain well within the crustal-seismic band, with magnitudes decreasing with depth. Since microstructural and material properties of the squeeze-flow process control the overall frequency

5.3.1 Attenuation in transitional zone

As discussed in section-5.1.2, the top of the transitional zone can be considered as an hydraulically-fractured formation in which a saturating fluid nears lithostatic pressure. Squeeze-flow is therefore applicable, but some allowance must be made for fluid-pressure exceeding lithostatic. I considered two scenarios: 1) fluid pressure is "normal", i.e. lithostatic or slightly smaller, to maintain chemical equilibrium; and 2) fluid pressure exceeds lithostatic, without reaching the tensile strength of the confining rock. The first case can be treated as above; however, to properly treat the latter, stress-strain laws for pre-stressed materials are required and these lead to very unwieldy problems to solve. Here, I consider an alternative which, although entirely phenomenological, accounts for the observation that seismic velocities significantly decrease in overpressured zones (e. g. Nur & Walder, 1990). The interpretation of such decrease is straightforward: the wave "sees" a reduced shear modulus caused by solid-solid contacts tending to separate due to overpressurised fluids.

Figure-5.10 illustrates the essence of my approach which uses the dual representation of a standard-linear solid (see Ben-Menahem & Singh, 1981). Both represen-

tations exhibit the same classical peaked $Q^{-1}(\omega)$ but differ in the manner in which the constituent moduli combine. For the normal pressure situation (bottom left of fig-5.10), solid-solid contacts can be expected to dominate squeeze-flow driving; therefore, it is logical to add moduli in series, as done in defining Q^{-1} in Eq-5.15. However, for the overpressured situation (bottom right of fig-5.10), fluid excess pressure can be expected to dominate and so it is logical to add moduli in parallel. This gives more weight to the mechanical effect of fluid by augmenting the fractured-formation compliance relative to "normal".

Figure-5.11 shows the result of doing so for the bottom part of the upper crust (10 — 15 km) parameterised in Tables 5.1 and 5.2. As expected both frequency signatures are identical; however, pressurised squeeze-flow dissipation is much larger than normal (by more than an order of magnitude, with peak $Q^{-1} \sim 1/5$). Although fluid reservoirs of the type modelled by Bailey (1990) are expected to be very thin (a few tens of meters at most according to him), fig-5.11 raises the interesting possibility that a good deal of surface $Q_t^{-1}(\omega)$ may result from a small zone of major dissipation at midcrustal level (cf. case-3, fig-2.5; chapter-2).

Squeeze-flow in top of transitional zone

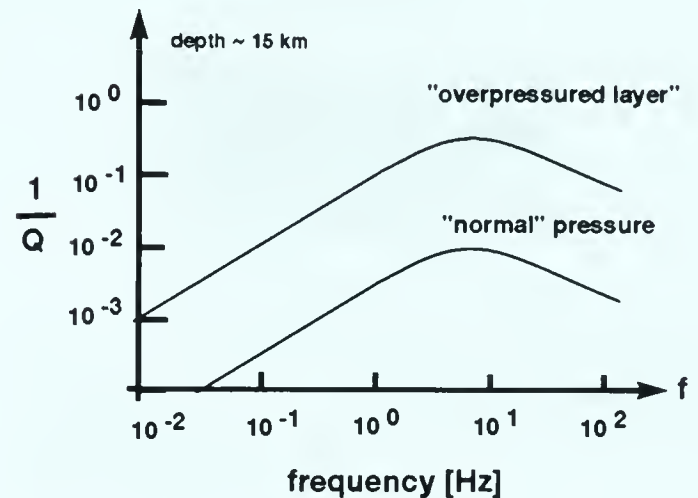


Figure-5.11 Squeeze-flow dissipation at 15 km in hydraulically fractured fluid reservoir. Frequency Q^{-1} signature is same for overpressured and normal conditions. Q^{-1} magnitude is much larger for overpressured fluids.

5.3.2 Attenuation in the lower crust

For the lower crust, assuming laterally homogeneous layers (e. g. Marquis & Hyndman, 1992), dissipation by squeeze-flow could in principle be estimated from combining the original sphere-pack model with smooth-surface contact-stiffnesses, since it is conceptually closer to the texturally equilibrated pore space inferred; however, smooth grain surfaces generate negligible dissipation by localised fluid-flow in analogous materials (Murphy et al., 1986). Therefore, the occurrence of squeeze-flow is unlikely since the basic contributing element — fracture roughness — is absent in the ductile regime. Another rejecting criterion is the rough estimate of the characteristic frequency of local fluid-flow (Eq-2.18) which, for expected material properties and inferred pore aspect-ratios for the lower crust, yield frequencies much larger than crustal-seismic (chapter-2).

Another method for examining dissipation from viscous motion of pore-fluids in lower crustal formations is to estimate Biot's bulk-flow effect, to find out whether it might dominate losses by localised fluid flow. Rasolofson (1991) produced plots of $Q^{-1}(\omega)$ comparing the Biot effect and a generalised standard-linear solid mechanism; some plots showed that, although Biot's effect occurs at higher frequencies, its low-frequency tail-end grazes the peak of the standard-linear solid mechanism. However, such frequency signature occurs only for highly permeable media, quite unlike the 10^{-18} — 10^{-19} m² permeabilities expected for the lower crust. Nevertheless, it is of interest to estimate the characteristic frequency of Biot's mechanism for lower-crustal layers. This frequency is approximately given by (e. g. Géli et al., 1987)

$$f_c \approx \frac{1}{2\pi} \frac{\Phi \mu}{K \rho_f} \quad [5.19]$$

Contact Stiffness of Saturated Rough Fractures

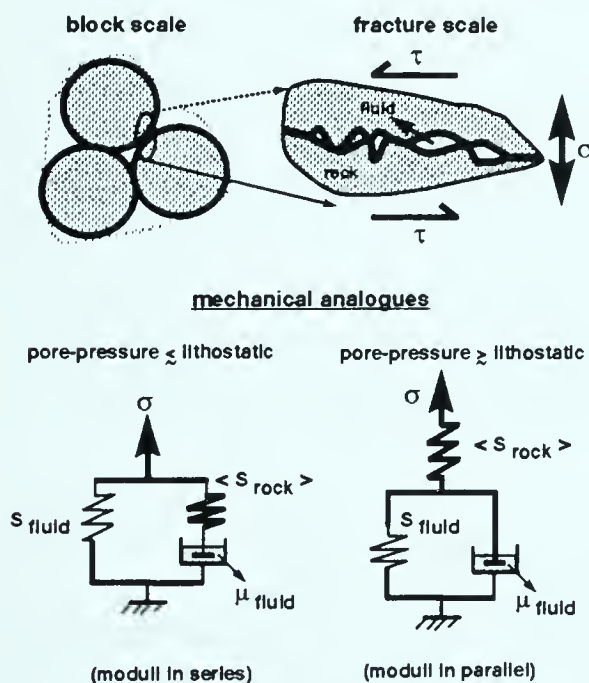


Figure-5.10 Top: assumed structure at top of transitional zone; fluid saturates rough interspace of hydraulic fractures. σ and τ compressional and shear stress, respectively. Bottom: two mechanical analogues for squeeze-flow dissipation; $\langle S_{rock} \rangle$, S_{fluid} and μ_{fluid} symbolise rock contact-stiffness, fluid stiffness, and viscosity, respectively; (left) fluid pressure slightly less than lithostatic, moduli add in series; (right) fluid pressure exceeds lithostatic, moduli add in parallel.

where ρ_f is fluid density. Using $\mu \approx 10^{-4}$ Pa-s for saline water still in the liquid state at 400 °C (Bodnar & Costain, 1991); $\Phi \approx 1\%$ for equilibrated porosity; $K \approx 10^{-18}$ m² for permeability; and $\rho_f \approx 1100$ kg/m³ for water, $f_c \approx 145$ MHz and so hopelessly removed from the crustal seismic band. Therefore neither squeeze-flow nor Biot's bulk flow mechanism seems to operate at seismic frequencies in the lower crust.

Adopting the viewpoint of Merzer & Klemperer (1992), that spatially overlapping isolated lamellae form a zig-zag electrical network whose effective resistivity gives that observed, suggests a source of scattering attenuation in the lower crust since these lamellae have dimensions nearing seismic wavelengths. Campillo & Paul (1992) examined the effect of such lamellae on synthesised codas of regional seismograms, by modelling lower-crustal reflectors as a finely layered structure of high impedance contrasts. Disregarding intrinsic attenuation, they found, by comparison with actual seismograms, that generated codas were much longer than observed (not a surprising result) and that scattered seismic energy did not account for the observed total attenuation. They suggested that scattering by randomly distributed lamellae instead of regularly layered ones could be the cause for the total attenuation signature.

I performed this suggested exercise by assuming that porous lamellae are randomly distributed in the lower crust and by using Sato's (1982) scattering theory for estimating $Q_{sc}^{-1}(\omega)$ for shear-waves [see, Eq-2.1, chapter-2]. Figure-5.12, inspired from Merzer & Klemperer's (1992) work, depicts the situation. For lamellae dimensions, I assumed a 100 m thickness and took a length of 6 km as a characteristic radius of inhomogeneities (this radius corresponds to the characteristic correlation length in Sato's theory). I assumed that the spatial distribution of lamellae is described by a Von Karmann autocorrelation function of order 1/3 and mean-square fractional velocity fluctuations of 0.001, roughly corresponding to a generous 3% impedance contrast as suggested by Sandmeier & Wenzel's (1990) shear-wave results. I also took an average shear-wave velocity of 4 km/s for the hydrated lower crust.

The result is shown in figure-5.13 where it is seen that scattering attenuation is frequency-dependent with a peak value of $\sim Q_{sc}^{-1} \approx 1/1000$; therefore much smaller than squeeze-flow Q^{-1} elsewhere in the crust. This low attenuation value is of the order often reported from inversion of $Q_t^{-1}(\omega)$ data (Cong & Mitchell, 1988).

Scattering by Brine-saturated Lamellae [Lower Crust]

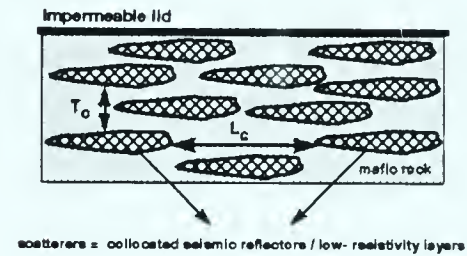


Figure-5.12 Geometrical attributes of seismic scatterers in lower crust. T_c is saturated lamellae thickness, L_c characteristic length. Lamellae assumed randomly distributed throughout lower crust.

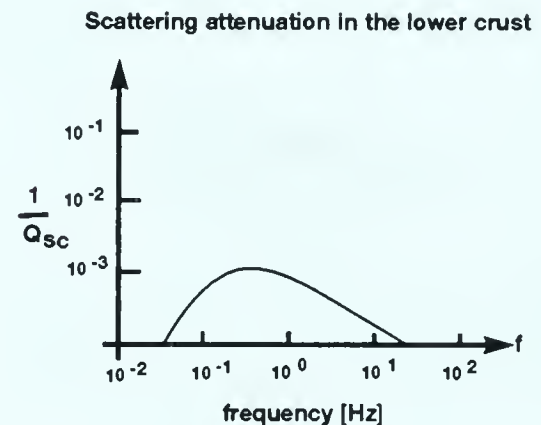


Figure-5.13 Scattering attenuation in shear for lower crust from randomly distributed fluid-filled km-scale lamellae

5.4 Conclusion

The purpose of this chapter was to estimate the depth-distribution of seismic attenuation in the crust, guided by an ubiquitously observed characteristic frequency round one Hz at which the total attenuation appears to peak within the crustal-seismic band 0.08 — 25 Hz. An operational framework was defined within which seismic attenuation in hydrated crust can be described, accordingly to the state of hydration at various thermodynamic conditions in the crust. A particular effort was directed at scaling the microscopic squeeze-flow mechanism, postulated to operate within rough-fracture interspace in brittle crust, to crustal-scale layers.

An effective-medium technique was established for estimating macroscopic properties from field-based observations in order to compute squeeze-flow $Q^{-1}(\omega)$ and associated dispersion $v(\omega)$. This technique is conceptually consistent with the fragmented nature of brittle crust, the natural roughness of crustal fractures, stress-strain laws for the mechanical behaviour of rough-surfaces in contact, hydraulic manifestations of the associated fracture-porosity, and with the theoretical premiss of linear viscoelasticity.

Biot's bulk-flow energy loss mechanism was estimated for the upper and lower crust and found negligible at seismic frequencies. Regarding the transitional crust as a network of sub-horizontal intersecting fractures saturated with a pore-liquid at either lithostatic or in excess of lithostatic pressure, squeeze-flow dissipation was estimated and found significant in the midcrust at seismic frequencies, much more so for overpressured layers. This result suggests that a pressurised layer, however thin, may have a significant effect on the total attenuation. Scattering attenuation by impedance contrasts characteristic of seismic reflectors / high-resistivity porous lamellae (Merzer & Klemperer's model) was estimated for the lower crust and appears to dominate over seismic energy losses by the dissipative mechanism.

Figure-5.14 summarises the results for the depth-distribution of seismic attenuation in the crust. Dissipation by squeeze-flow seems predominant in the upper and middle crust, with apparently robust characteristic peaks well within the crustal seismic band. As such, the squeeze-flow mechanism developed in this thesis constitutes the only seismic dissipation model which respects available constraints on the hydraulic and heterogeneous nature of the crust — whilst producing peak dissipations in the seismic band. It is interesting to note that internal Q^{-1} characteristic frequencies fall to the right of the roughly one Hz total Q_t^{-1} peak reported for the crust (fig-2.1/2.2, chapter-2). This may result from a continual shifting of characteristic frequencies to higher values as depth (and temperature) increases in the brittle crust; this was well-illustrated in case-2 of figure-2.5 (chapter-2) where the apparent peak was much lower than internal ones (see also fig-2.4). It may also reflect that the relatively "pure" fluid properties used to estimate internal Q^{-1} underestimate the *in-situ* fluid viscosity; additional molecular compounds, likely present in crustal fluids, logically contribute more viscosity and so internal peak Q^{-1} 's can be expected to shift to lower frequencies, as suggested by the degree of sensitivity of the squeeze-flow dissipative cell to viscosity changes (see fig-4.13, chapter-4).

It is clear that computing squeeze-flow Q^{-1} from the method described here leads to model-dependent results, especially regarding macro and micro structural parameters. This is a common feature of all models of dissipation by localised viscous flow. That this approach yields results consistent with observation seems satisfactory; however, it is arguably desirable to minimise the number of model parameters and cast the phenomenon in less assuming terms. Work along this line of thought is only beginning. Dvorkin & Nur (1993) recently unified both

Seismic attenuation in hydrated crust

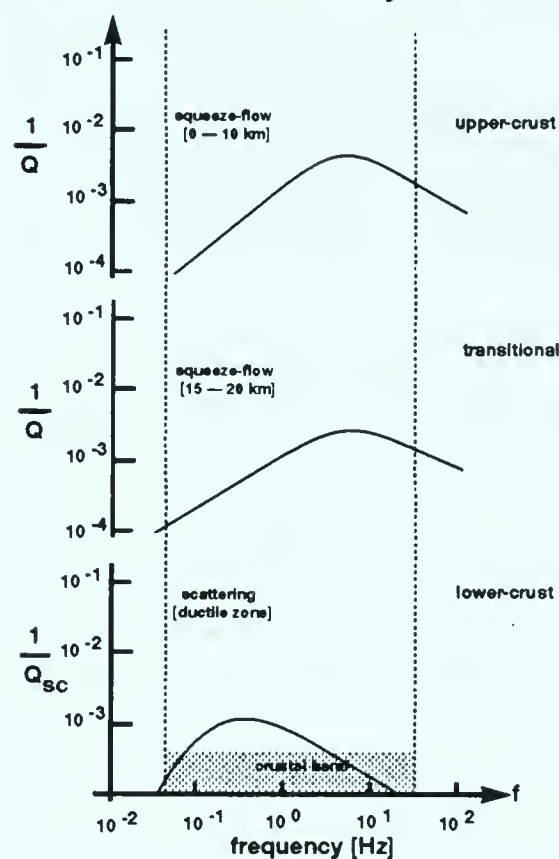


Figure-5.14 Composite attenuation map for hydrated crust at seismic frequencies. Dissipation by squeeze-flow appears predominant in upper and transitional crust. Scattering attenuation in lower crust appears significant.

squirt-flow and Biot mechanisms under a single set of governing equations, by invoking (as well) the concept of a "characteristic crack". This is a first attempt at relating these mechanisms which are conventionally treated separately. Their model requires fewer parameters than used here and opens the way to investigate more substantial links between microscopic and macroscopic scales.

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Chapter-6: Conclusion

By exploiting the idea that localised viscous flow of interstitial aqueous fluids preferentially occurs within the interspace of rough-walled natural fractures, I re-established the laboratory-inferred mechanism of "squirt-flow" as a plausible cause for the characteristic frequency-dependent attenuation reported from crustal seismic-waves in the bandwidth 0.08 — 25 Hz (chapter-2). I elaborated a comprehensive theory for compressible viscous squeeze-flow of thin fluid-films in a low aspect-ratio crack which provides solid mathematical and physical support to previous *ad hoc* models. The ensuing dimensionless compressible-flow governing equation allows to discriminate flow regimes from dimensionless constants prior to solving for boundary conditions and suggests ways of attacking problems of non-linear dissipation (chapter-3).

Using idealised, yet realistic boundary conditions for cracks typical of natural fractures, I found analytic solutions for the complex mechanical stiffness of a characteristic crack saturated with a compressible aqueous fluid, from which the physics of dissipation by oscillatory squeeze-flow — within the crustal-seismic band — was made apparent. This stiffness is sensitive to crack thickness, aspect-ratio, and local porosity, and to fluid viscosity and temperature. I showed that dissipation by squeeze-flow is significant for crustal conditions and produces a robust peak which remains within the crustal-seismic band down to depths possibly extending to 20 km (chapter-4).

Within the framework of a schematic model of a three-layer hydrated crust, constrained by geological and geophysical inferences, I calculated dissipation/dispersion pairs for the two topmost brittle layers by extending an effective-medium scheme proposed by Winkler (1983) and Palciauskas (1992) for granular rock-samples (chapter-5). This extended technique is valid inasmuch as crustal Poisson ratios are significantly less than 0.25. The block structure of fractured crust is explicit in the calculations and crustal permeability and porosity, consistent with the peculiar rough microstructure of natural fractures and their mechanics, are recognised. Together with the microscopic-scale properties of the crack stiffness of chapter-4, the dissipation/dispersion pairs found for these layers are sensitive to crustal-scale porosity, block-size and block-contact mechanical deformability. For the bottom layer, characterised by equilibrium pore-geometry in a ductile regime as inferred from concomitant seismic reflectors/low resistivity zones, squeeze-flow appears insignificant, although seismic scattering may be important.

Therefore, to the question posed in section-1.2, namely:

Can the bell-shaped frequency-dependent seismic attenuation reported for the lithosphere result from preferential dissipation by viscous flow of interstitial fluids with characteristic frequencies within the band of high-frequency seismic-waves?

the answer is a qualified yes, since dissipation by squeeze-flow, as modelled here, was found significantly operative only in the brittle part of the Earth's crust. Inasmuch as scattering is weak and effectively frequency-

independent in the crustal-seismic band proper, dissipation by squeeze-flow of crustal-fluids within fracture interspace is more than plausible, as attested by the consistency obtained from the integration of a fair body of independent inferences on the hydraulic nature of the crust and from the calculated magnitude and frequency dependence signature of Q^{-1} which compares well with high-frequency attenuation data, albeit nominally. Given that the squeeze-flow model developed here is the only one currently consistent with crustal hydraulic constraints, it may be of some help to further advance seismic-attenuation research and its applications.

With respect to proportioning dissipation and scattering in seismic-attenuation measurements, forward computations based on the approach taken in chapter-5 can now be made to independently estimate dissipative Q^{-1} , thereby allowing to concentrate on scattering Q_{sc}^{-1} (the reverse is the common practice). This seems in need presently, as Wennerberg (1993) recently pointed-out that one of the most comprehensive scattering theories (Zheng, 1991) yields negative values of coda-wave attenuation when dissipation is disregarded, in contradiction with observation (when dissipation is considered, coda-wave Q_c^{-1} appears dominated by it).

Given a well-constrained tectonic area, where crustal fluids are suspected, attenuation data spanning a wide enough bandwidth could be synthesised by trial-and-error from specified depth-distributions of squeeze-flow $Q^{-1}(\omega)$. This can be done by applying the correspondence principle to formulate the problem. A successful fit could then be interpreted in terms of the crustal-fluid attributes most active in the squeeze-flow mechanism. From the inverse problem viewpoint, squeeze-flow dissipation/dispersion pairs can be incorporated into Lee & Solomon's (1978) simultaneous inversion scheme for mapping frequency-dependent $Q^{-1}(\omega)$ in the Earth, although resolution and especially data paucity are likely to be problematic. It is worthwhile noting that the squeeze-flow model is not restricted to crustal-scale problems; the model remains of interest in so far as an hydrated rock-formation of smaller-scale has rough-walled fractures and exhibits notable frequency-dependent dissipation in the sampling bandwidth.

Finally, it is worth mentioning that the calculations presented in chapter-5 for shear dissipation (Q_s^{-1}) can easily be extended to dilatational dissipation (Q_p^{-1}), following the squirt-flow modelling of Murphy et al. (1986), but using the crack stiffnesses developed in chapter-4. This may be of interest to attempt an explanation of recently reported high Q_p^{-1} / Q_s^{-1} ratios above 1 Hz for several tectonic areas (Yoshimoto et al., 1993). Although these authors contend that differential scattering of P/ S converted waves is the cause, it may also be due to squeeze-flow of a gas-liquid mixture, CO_2 vapour being a candidate for these areas.

6.1 References

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Appendix

Insertion of the normalised variables of Table-3.1 into the operator D/Dt defined by Eq-3.6 yields

$$\overline{\left(\frac{D}{Dt}\right)} = \omega \left[\frac{\partial}{\partial T} + u_k \frac{\partial}{\partial X_k} + w \frac{\partial}{\partial z} \right] + \frac{V}{S} u_k \frac{\partial}{\partial X_k} \quad [A.1]$$

where the bar denotes normalisation. Applying this result to the dilation Δ defined by Eq-3.9 yields, with $\rho = \rho_0 \bar{\rho}$,

$$\bar{\Delta} = \frac{\omega}{\bar{\rho}} \left[\frac{\partial \bar{\rho}}{\partial T} + u_k \frac{\partial \bar{\rho}}{\partial X_k} + w \frac{\partial \bar{\rho}}{\partial z} \right] + \frac{V}{S} u_k \frac{\partial \bar{\rho}}{\partial X_k} \quad [A.2]$$

Multiplying A.2 by $(\lambda + \mu)$ and rearranging terms gives the compact form

$$(\lambda + \mu) \bar{\Delta} = \mu \left(\omega \Delta_s + \frac{V}{S} \Delta_l \right) \quad [A.3]$$

where Δ_s and Δ_l are defined as

$$\Delta_s = \frac{(1 + \lambda/\mu)}{\bar{\rho}} \left[\frac{\partial \bar{\rho}}{\partial T} + u_k \frac{\partial \bar{\rho}}{\partial X_k} + w \frac{\partial \bar{\rho}}{\partial z} \right] \quad [A.4]$$

$$\Delta_l = \frac{(1 + \lambda/\mu)}{\bar{\rho}} u_k \frac{\partial \bar{\rho}}{\partial X_k} \quad [A.5]$$

Normalising the kinematic constraints Eq-3.15 and Eq-3.16 gives

$$\omega_a W = \omega \frac{\partial \bar{a}}{\partial T} + \frac{V}{S} u_k \frac{\partial \bar{a}}{\partial X_k} \quad [A.6]$$

$$\omega_b W = \omega \frac{\partial \bar{b}}{\partial T} + \frac{V}{S} u_k \frac{\partial \bar{b}}{\partial X_k} \quad [A.7]$$

where $\bar{a} = S_a / h_0$ and $\bar{b} = S_b / h_0$.

To normalise the continuity equation it is best to first simplify its dimensional expression, Eq-3.7. Developing this expression into its components gives

$$\frac{\partial \rho}{\partial t} + v_j \frac{\partial \rho}{\partial x_j} + \rho \frac{\partial v_j}{\partial x_j} = \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_j)}{\partial x_j} = 0 \quad [A.8]$$

Making explicit velocity component v_3 and letting $l = \{1, 2\}$ results in

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_l)}{\partial x_l} + \frac{\partial (\rho v_3)}{\partial x_3} = 0 \quad [A.9]$$

Integrating A.9 from $S_a \equiv a$ to $S_b \equiv b$, i-e over the fluid-film's thickness, yields

$$h \frac{\partial \rho}{\partial t} + \int_a^b \frac{\partial (\rho v_l)}{\partial x_l} dx_3 + \int_a^b \left(\rho \frac{\partial v_3}{\partial x_3} + v_3 \frac{\partial \rho}{\partial x_3} \right) dx_3 = 0 \quad [A.10]$$

where $\partial \rho / \partial x_3 \approx 0$ since the vertical pressure gradient is negligible to first order in perturbation parameter ϵ (Eq-3.32). Using this result, and applying Leibnitz rule to the second term of A.10, leads to

$$h \frac{\partial \rho}{\partial t} + \rho \left\{ \left({}_b V_3 - {}_b V_l \frac{\partial S_b}{\partial x_l} \right) - \left({}_a V_3 - {}_a V_l \frac{\partial S_a}{\partial x_l} \right) \right\} = - \frac{\partial}{\partial x_l} \left(\rho \int_a^b v_l dx_3 \right) \quad [A.11]$$

where ρ has been factored-out of the integral, as justified by Eq-3.32. The differences in parentheses in A.11 can be succinctly expressed using the kinematic constraints Eq-3.15 and Eq-3.16. Thus, the reduced form of the dimensional continuity equation takes the compact form

$$\frac{\partial (\rho h)}{\partial t} + \frac{\partial}{\partial x_l} \left(\rho \int_a^b v_l dx_3 \right) = 0 \quad [A.12]$$

Insertion of the appropriate normalised variables of Table-3.1 into A.12 yields the non-dimensional form of the continuity equation given by Eq-3.34.

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